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The Eötvös Torsion Balance Method of Mapping Geologic Structure

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The Eötvös Torsion Balance Method of Mapping Geologic Structure

By DONALD C. BARTON,* HOUSTON, TEXAS

(New York Meeting, February, 1928)

THE theory of gravitation is based on Newton's law that any two bodies exert a mutual attraction which is proportional to the product of their masses and inversely proportional to the square of the distance between them; *i. e.*,

$$A = K \frac{Mm}{R^2} \quad (1)$$

where K is a constant, M and m are the respective masses of the two bodies, and R the distance between them.

In the earth's gravitational system, the earth is one of the bodies; the other may be any body within or without the earth. In theoretical discussions and calculations in regard to gravity, that other body usually is taken as a unit of mass: in the C. G. S. system, a body of one gram in mass.

GRAVITY AND LEVEL SURFACES

Gravity is defined in geophysics as the force of attraction exerted by the earth, towards itself, on a body of unit mass. The intensity of gravity is usually expressed in dynes of force or in centimeters of acceleration. In geophysical work in America, most commonly it is expressed in dynes (per gram) with the "per gram" understood and not directly expressed. In geophysical work in Europe, most commonly it is expressed in centimeters per second per second (cm./sec.²) and less commonly merely as centimeter-gram-second units (C. G. S.). At sea level, the intensity of gravity amounts very nearly to 980 dynes.

The vertical is the direction along which the attraction of gravity is exerted at any point. In most cases, it is very nearly, but not quite, perpendicular to the earth's surface. The path that would be taken by a body falling freely through space toward the earth is vertical at each point and may be spoken of also as "the vertical," or as the line of the vertical, or as a line of force of gravity. At the earth's surface it commonly is very nearly, but not quite, a straight line.

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A level surface¹ is a one that is everywhere perpendicular to the vertical. There are an infinite number of level surfaces, but for purposes of brevity in discussion and clarity and ease in graphic representation, an arbitrary set of level surfaces at some arbitrary interval often is used as though only these surfaces existed. The surface of a body of water at rest and acted on only by gravity is a level surface. A level surface is not a plane and it is not a surface of equal gravity.²

A horizontal surface, in contrast to a level surface, is a plane surface tangent to the level surface and perpendicular to the vertical at some observation point or some other point which for the moment is being used as the center of reference.

If the earth were a homogeneous sphere at rest, gravity would be the same everywhere over the surface of the earth and would vary only radially; the lines of force of gravity would be radial, and the level surfaces would be spherical and concentric with the earth's surface. But the earth is not at rest; it is not a sphere; and its outer crust is in no way homogeneous. On account of the rotation of the earth, centrifugal force tends slightly to counteract the earth's attraction. The effect is at a maximum at the equator and decreases to zero at the north and the south poles. As the earth is a spheroid flattened at the poles, and therefore the radial distance of the surface at the poles from the center of the earth is less than at the equator, the force of attraction of the earth is at a maximum at the poles and at a minimum at the equator. As the result of those two factors, the value of gravity at sea level is at a maximum at the poles and a minimum at the equator. The increase of the value of gravity northward in the northern hemisphere is known in torsion balance work as the "Normal Northward Gradient" and amounts to 7 Eötvös units (or 7×10^{-9} dynes) at latitude 30. It is a distinctly appreciable quantity and has to be allowed for in the calculations. The level surfaces are warped concomitantly and the amount of the warping is a function of latitude. It is of a very appreciable magnitude and the "normal differential curvature," as it is called in Eötvös torsion balance work, has to be allowed for in connection with the "differential curvature or R " values, to be described later.

The earth's crust is distinctly inhomogeneous. The continents are supposed to be composed of lighter rocks than are the ocean basins. Plutonic igneous complexes and metamorphic complexes are heavier than most masses of sedimentary rocks. Limestone and anhydrite are heavier than other sediments, and the older sands and shale tend to be

¹ "Niveau surface" in some of the torsion balance literature; "equipotential surface" of geophysics and of the potential theory.

² The level surface being an equipotential surface, it is a function of $\frac{1}{R}$, and a surface of equal gravity is a function of $\frac{1}{R^2}$ where R is the distance to the attracting body.

heavier than the younger. Horizontal uniformity of distribution of sediments of uniform density is destroyed by diastrophism. The horizontal distribution of mass in the earth's crust within a few miles of the surface, therefore, is very irregular; bodies of rock of high specific gravity rise into or beside bodies of rock of lesser density and the magnitude of those bodies ranges from that of a mountain system down to that of a glacial boulder. Topographic relief also causes a horizontal opposition of rock of relatively high specific gravity against air of relatively no specific gravity. This irregularity of the horizontal distribution of mass causes a deformation in the gravitational system. A body of density

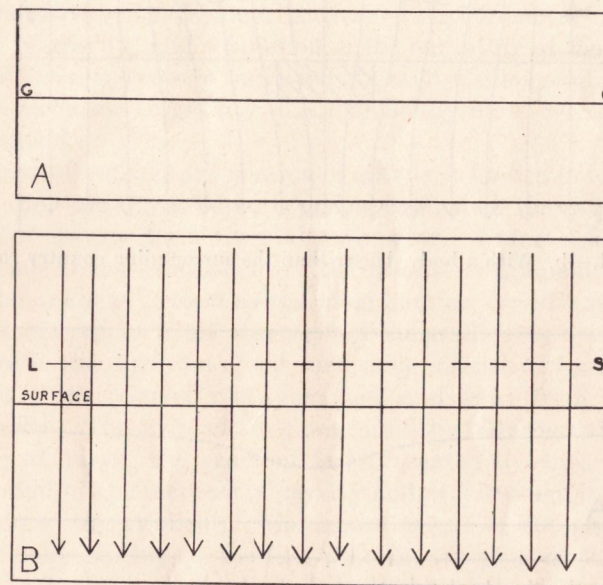


FIG. 1.—DIAGRAMMATIC SKETCHES SHOWING A SMALL SECTION OF THE EARTH'S CRUST AND GRAVITY RELATIONS.

a. With a homogeneous distribution of mass.

greater than the average will exert a definite gravitational attraction approximately toward its center of gravity. The intensity of the attraction will be proportional to the excess of mass. There is no repulsion possible in the gravitative system, but the practical effect of a body less dense than the average is the same as if the body exerted a force of repulsion proportional to its deficiency of mass. Gravity at any point is the three dimensional vectorial sum of the normal gravity and of the respective positive and "negative" forces of attraction of the various bodies with excess or deficiency of mass.

As the result of the irregular distribution of mass in the earth's crust, the intensity of gravity locally is at a maximum over the areas of excess

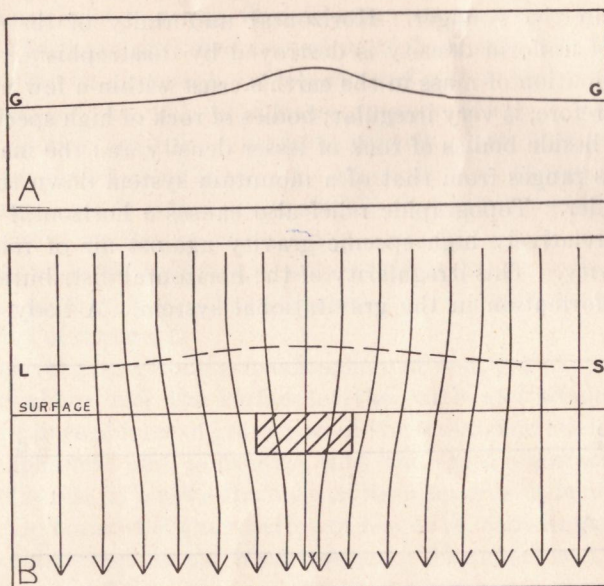


FIG. 1.—b. With a body denser than the surrounding country rock.

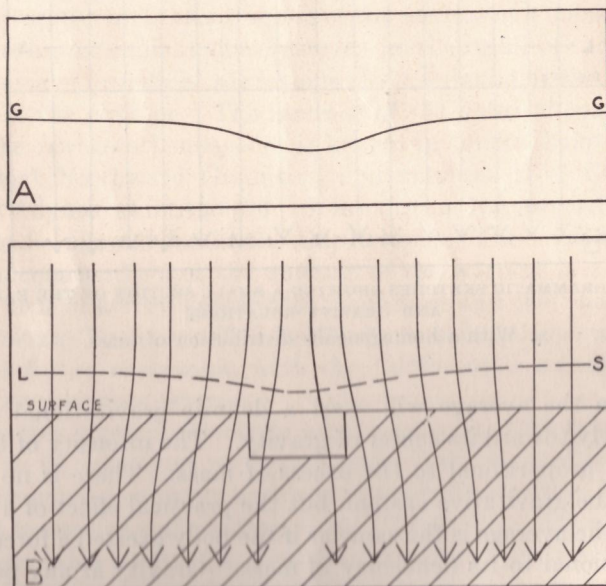


FIG. 1.—c. With a body lighter than the surrounding country rock.

gg = profile of gravity.
 LS = level surface.
 Arrows = lines of the vertical.

of mass and at a minimum over the areas of deficiency of mass; the lines of the vertical tend to crowd from the bodies deficient in mass into the bodies with excess of mass—that is, the vertical is deflected toward the excess of mass and away from the deficiency of mass; the level surfaces are warped up, convexly, over the areas of excess of mass and are warped downward, concavely, over the areas of deficiency of mass. (Fig. 1.)

ELEMENTS MEASURED BY THE TORSION BALANCE

The torsion balance measures directly two elements of that deformation in the gravitational system: the rate of the horizontal variation of the intensity of gravity, and a function of the curvature of the level surface. Indirectly from the rate of variation of gravity, it is possible to calculate the total variation of gravity, if a sufficiently close net of stations has been occupied.

The rate of horizontal variation of gravity is known in torsion balance work as the “gravity gradient” and is defined as the difference in the intensity of gravity per horizontal centimeter. Unless definite specification is made that a component of the maximum gradient is meant, the term “the gradient” refers to the maximum or “total” gradient. The convention in regard to the direction of the gradient is that the gradient is positive in the direction of the increasing intensity of gravity; that is, it is toward the relatively heavier mass and away from the relatively lighter mass. The symbol Gr g was used by Eötvös to designate the gradient; on maps, the gradient is represented by an arrow flying in the direction of the increase of gravity and proportional in length to the magnitude of the gradient. The measurement of the gradient by the torsion balance is due to the fact that the warping of the vertical, or what is the same thing, the divergence of the level surfaces, produces at each weight of the balance a small horizontal component of gravity, which is equal to the horizontal gradient of gravity.

The differential curvature that is measured by the Eötvös torsion balance is not the actual curvature of the level surfaces but is a function of the difference in the curvature of the level surface in the directions of greatest and least curvature. Any level surface taken over a considerable area is a very irregularly warped surface and can not be represented by a simple mathematical surface, but within the small area of the torsion balance, it can be represented by either an ellipsoidal or a saddle-shaped surface of very large radius of curvature. Within the area of the torsion balance, the trace of that surface in vertical planes approximates a small portion of a very large circle. The differential curvature is the difference between the reciprocals of the radii of curvature of that surface in the vertical planes in the directions of the greatest and the least curvature.

The curvature value, or the R value of the literature which was used by Eötvös and is used in most work with the torsion balance, is gravity times that difference between the reciprocals of the radii of greatest and least curvature (minimum and maximum radii of curvature); that is:

$$R = g \left(\frac{1}{r_{\min}} - \frac{1}{r_{\max}} \right). \quad (2)$$

The measurement of R by the torsion balance is due to the fact that the warping of the level surfaces produces a small component of gravity at each weight of the balance. The resulting moment of torsion is a function of R and the azimuth of the balance beam. The orientation of R by convention is taken as that of the axis of algebraically minimum curvature (maximum radius of curvature) and the azimuth of that axis is designated by the symbol λ . On maps, R is represented graphically by a line oriented with an azimuth of λ and with a length proportional to the magnitude of R .

TORSION BALANCE UNITS OF MEASUREMENT

The unit of measurement for the gradient in work with the Eötvös torsion balance is 1×10^{-9} dynes (per gram) per horizontal centimeter. That unit is coming to be called an "Eötvös" or an "Eötvös Unit." A gradient of 1 E means an increase in the intensity of gravity per horizontal centimeter of about one thousand-billionth (1×10^{-12} th) the value of gravity. The values of the gradient actually measured in the field range most commonly from 5 to 30 E. The mean maximum value for the gradient observed in work with which the writer was connected is about 150 E.

The unit of measurement for the differential curvature, strictly speaking, should be in units of 1×10^{-12} radians per centimeter, as the significance of the magnitude lies in its indication of the difference of the warping of the level surface in different directions. The R that is actually used in practice, however, is gravity times that difference, and is measured in terms of 1×10^{-9} dynes per gram per centimeter (Eötvös units). But as the reciprocal of the value of gravity in most places is within a few per cent. of 1×10^{-3} and as the field measurements of R are not accurate within a few per cent., the numerical value of R in terms of 1×10^{-9} dynes per gram per centimeter (Eötvös units) is the same for most practical purposes as the numerical value of the differential curvature in terms of 1×10^{-12} radians per centimeter. The values of the differential curvature actually observed in the field commonly range from 5 to 50 Eötvös units. The mean maximum value for the largest observed values of R is about the same numerically as that for the gradient: 150 Eötvös units. Large values for R are very much commoner than large values for the gradient.

WORKING SYSTEM OF EÖTVÖS TORSION BALANCE

The working system of the Eötvös torsion balance consists essentially of a torsion wire suspended at its upper end and carrying at its lower end a freely swinging horizontal aluminum bar. A gold or platinum weight is fastened at one end of the bar and an equal weight is suspended by a fine wire from the other end of the bar. If a horizontal moment of rotation acts on the weights, the bar tends to rotate and to twist the torsion wire; the latter tends to resist the torsion; and the balance bar comes to rest when the resistance of the wire to torsion is equal to the torque.

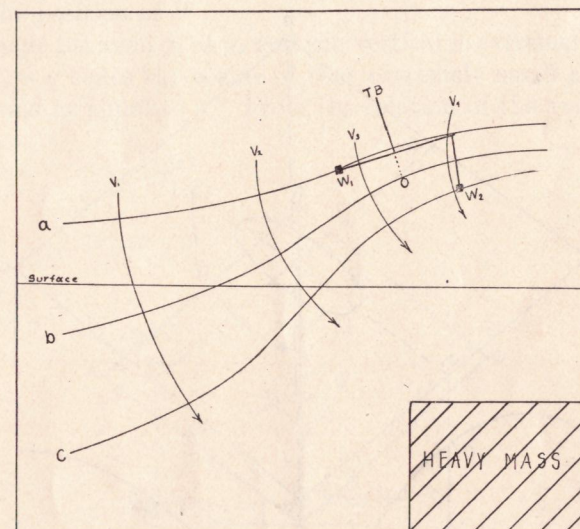


FIG. 2.—EÖTVÖS TORSION BALANCE IN A VERTICAL SECTION THROUGH A GRAVITATIONAL FIELD.

The phase of the curvature of the level surfaces is represented more in detail by Fig. 3 and the phase of the curvature of the vertical is represented by Fig. 5.

V_1 to V_4 = lines of the vertical; a, b, c = level surfaces; TBW_1W_2 = Eötvös torsion balance; W_1 , upper weight, W_2 , lower weight.

If the rotation of the wire is small, the resistance of the wire is directly proportional to the angular amount of the torsion. The torque necessary to twist the torsion wire through any given angle can be determined in the laboratory and a coefficient of torsion can be calculated for the wire. If an unknown torque causes a rotation of the bar and if the torsion coefficient of the torsion wire is known, the magnitude of the torque can be determined by measuring the angle through which the balance bar has turned. The only practical improvement that has been made in Baron Eötvös' original design of the essential working system of the torsion balance is the Z-shaped balance bar introduced by Schweydar of Berlin. The balance bar has roughly the shape of a "Z" on its side in a vertical plane; and the weights are fastened rigidly to the ends of the upper and

lower arms of the "Z;" the torsion wire is thereby lowered without lowering the center of gravity of the instrument. Various other modifications of the balance have been designed and are being worked on, but no other modification of the essential working system of the balance has been perfected.

MEASUREMENT OF THE DIFFERENTIAL CURVATURE

In its measurement of the differential curvature, the Eötvös torsion balance virtually is a special form of a much older type, the Coulomb

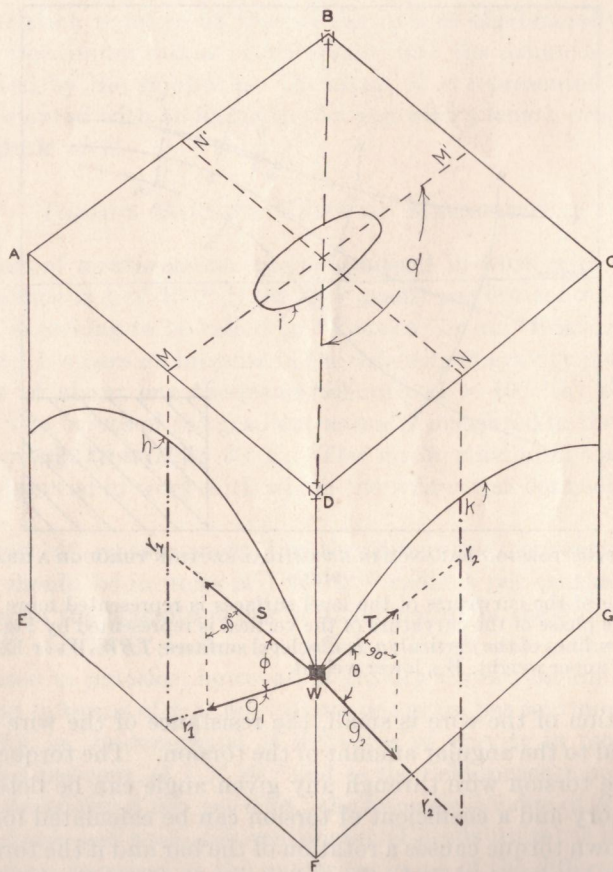


FIG. 3.—DIAGRAMMATIC BLOCK DIAGRAM OF THE SPACE SYSTEM AROUND AN EÖTVÖS (OR COULOMB) TORSION BALANCE, REPRESENTING THE CURVATURE OF THE LEVEL SURFACES.

balance; in which both weights are at the ends of a straight bar and therefore in the same horizontal plane. The theory of the measurement of the differential curvature is as follows:

In Fig. 3, let i be the trace of a level surface in the horizontal plane $ABCD$; let $DCGF$ be the perpendicular plane parallel to the major axes, MM' , of that level surface; and $ADFE$ be the corresponding plane parallel to the minor axes NN' . Let k and h be the respective traces of that level surface in $DCGF$ and $ADFE$, BD be the projection on $ABCD$ of the balance beam, l be the half-length of the balance beam, W be one of the weights of the balance and be in the level surface, hik , m be the mass of the weight, and α be the angle of azimuth of the balance beam. Let r_1 and r_2 be the radii of h and k respectively, and g_1 and g_2 be the projections respectively on $ADFE$ and $DCGF$ of an arrow representing the value of gravity at the position of W .

As the angle between g_1 or g_2 and the vertical is extremely small, and as g_1 (or g_2) = g times the cosine of that extremely small angle, g_1 and g_2 may be used as equal to g . From the relation of the vertical to any

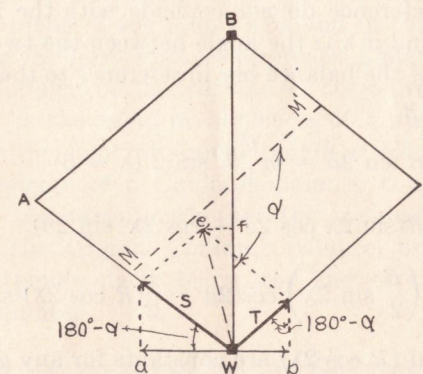


FIG. 4.—REPRESENTATION OF THE SITUATION IN THE HORIZONTAL PLANE THROUGH THE WEIGHT W OF FIG. 3.

level surface, g_1 and g_2 are perpendicular to h and k respectively and therefore are no longer perpendicular to the horizontal plane, and each has a horizontal component, S and T respectively.

Remembering the assumption that the profile of the level surface is the small portion of a very large circle,

$$S = g \cos \phi = g \cdot \frac{MD}{r_1} = g \cdot \frac{1}{r_1} l \sin \alpha \quad (3)$$

$$T = g \cos \theta = g \cdot \frac{ND}{r_2} = g \cdot \frac{1}{r_2} l \cos \alpha \quad (4)$$

Let Fig. 4 represent the horizontal plane through the position of W ; aW and bW will then be the components of S and T respectively, acting

perpendicularly to the balance bar, and their difference times (ml) will be the torque F tending to rotate the balance bar. Then,

$$aW = S \cos (180 - \alpha) - g \frac{l}{r_1} \sin \alpha \cos \alpha \quad (5)$$

$$bW = T \sin (180 - \alpha) = g \frac{l}{r_2} \cos \alpha \sin \alpha$$

$$F = (ml)(bW - aW) = (ml)lg \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \frac{1}{2} \sin 2\alpha \quad (6)$$

The magnitude $\left(\frac{1}{r_2} - \frac{1}{r_1} \right)$ is a constant for any given level surface and the magnitude $g \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$ is the R value or "horizontal directive force" (Richtkraft) of Eötvös. Geometrically F is represented by ef . The line eW is the horizontal projection of g at W , and ef is the component of eW perpendicular to the balance bar.

If the axes of reference do not coincide with the major and minor axes of curvature, and if λ is the angle between the two sets of axes and if β is the azimuth of the balance bar in reference to the axes of reference,

$$\alpha = \lambda - \beta \quad (7)$$

$$F = m \frac{l^2}{2} R \sin 2\alpha = m \frac{l^2}{2} R \sin 2(\lambda - \beta) \quad (8a)$$

$$= m \frac{l^2}{2} R (\sin 2\lambda \cos 2\beta - \cos 2\lambda \sin 2\beta) \quad (8b)$$

$$= ml^2 \left[\left(\frac{R}{2} \sin 2\lambda \right) \cos 2\beta - \frac{1}{2} (R \cos 2\lambda) \sin 2\beta \right] \quad (8c)$$

$\left(\frac{1}{2} R \sin 2\lambda \right)$ and $(R \cos 2\lambda)$ are constants for any given level surface and for any given set of axes. $(R \cos 2\lambda)$ can be shown geometrically to represent $\left(\frac{1}{r_x} - \frac{1}{r_y} \right)$ where r_x and r_y are the radii of curvature of the trace of the level surface in the vertical planes of the Y and the X axes respectively. There is no simple graphical representation of $\left(\frac{1}{2} R \sin 2\lambda \right)$. By differential geometry it is possible to show that:

$$\frac{1}{2} R \sin 2\lambda = \frac{d^2 U}{dx dy} \quad \text{and} \quad -R \cos 2\lambda = \left(\frac{d^2 U}{dy^2} - \frac{d^2 U}{dx^2} \right)^3$$

³ The terms $\frac{d^2 U}{dx dz}$, $\frac{d^2 U}{dy dz}$, $\frac{d^2 U}{dx dy}$, and $\left(\frac{d^2 U}{dy^2} - \frac{d^2 U}{dx^2} \right)$ often shortened in torsion balance work to U_{xz} , U_{yz} , U_{xy} ($U_{yy} - U_{xx}$) or U_{Δ} , are the shorthand symbols of calculus for those and two other quantities to be discussed shortly where U is the Newtonian gravitational potential. Any one approaching the theory from the geometrical side and not understanding the calculus of the potential function has to accept them merely as arbitrary symbols.

From those relations:

$$R^2 \sin^2 2\lambda + R^2 \cos^2 2\lambda = 4U_{xy}^2 + U_{\Delta}^2 \quad \text{and} \quad R = \sqrt{4U_{xy}^2 + U_{\Delta}^2} \quad (9a)$$

$$-\frac{R \sin 2\lambda}{R \cos 2\lambda} = \frac{2U_{xy}}{-U_{\Delta}} \quad \text{and} \quad \tan 2\lambda = -\frac{2U_{xy}}{U_{\Delta}} \quad (9b)$$

As β is known, U_{xy} and U_{Δ} can be calculated from equation (8) if at least two independent values of F have been obtained by observation. The values of R and λ can then be calculated by the formulas of equation (9).

The Eötvös torsion balance becomes a special type of the Coulomb balance if the assumption is made that within the very small area of the working system of the Eötvös torsion balance the curvature of all level surfaces is identical. The upper level surfaces in general will tend to have slightly flatter curvature than the lower level surfaces, but the difference between the curvature of the upper level surfaces and the lower level surfaces is very small compared with the difference between the curvature in the plane of the major axis and in the plane of the minor axis except where the level surfaces approach a plane or spherical form. The vertical difference in the curvature, therefore, can be neglected. If the curvature is the same at all levels of a system, the horizontal directive force is the same at similar points at all levels, and the force acting on the weights of a Coulomb balance is the same within the accuracy of the instruments whether the weights are at the ends of the beam, as in the original Coulomb balance, or one of the weights is suspended perpendicularly below the end of the beam, as in the Eötvös torsion balance.

MEASUREMENT OF THE GRADIENT

The great contribution made by Baron Eötvös was in showing that if the weights of the Coulomb balance are suspended in two level surfaces, one lower than the other, the balance measures the horizontal gradient of gravity as well as the differential curvature. The use of the Eötvös torsion balance to measure that gradient depends on the two theorems: (1) That if there is a horizontal gradient of gravity, the line of the vertical is curved; and (2) That if the line of the vertical is curved, there is at each weight of the balance a small horizontal component to gravity that is equal to the horizontal gravity gradient; also, as with the measurement of the differential curvature, on the assumption that as the area of the torsion balance system is very small compared to the distance to the attracting system, the variation of gravity is uniform within the torsion balance system.⁴

⁴ For experimental proof of the justification of this assumption, see, Karl Mader: Zur Verwendung der Drehwaage von Eötvös bei nahen grossen Massen. *Sitzungsber. d. Akad. d. Wissenschaft in Wien, Math.-Naturw. Klasse Abt. IIa* (1924) **133**, Pts. 3 and 4.

The proof of the first theorem is as follows: In Fig. 5, let AA' and BB' be any two level surfaces; PQ and $P'Q'$ be any two lines of the vertical- PP' being their intersections with AA' and QQ' their intersections with BB' ; dd' the distances respectively between PQ and $P'Q'$; g be the value of gravity along PQ , and $(g + \Delta g)$ the value of gravity along $P'Q'$. By the nature of the relation of the vertical to all level surfaces, PQ and $P'Q'$ must be perpendicular to AA' and BB' . Then:

1. If PQ and $P'Q'$ are straight lines, AA' must be parallel to BB' and d must equal d' ; or

2. If PQ and $P'Q'$ are curved, AA' and BB' are not parallel and d is greater or less than d' .

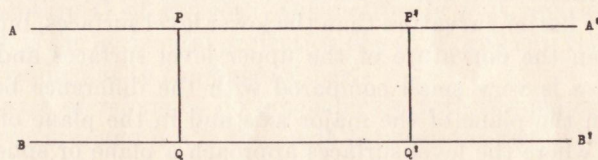


FIG. 5.—LEVEL SURFACES FOR PROOF OF FIRST THEOREM FOR USE OF EÖTVÖS BALANCE IN MEASURING GRADIENT.

The work done, if a unit of mass is moved from Q to P' , will be the same whether the mass is moved along $QP - PP'$ or $QQ' - Q'P'$ (no work is done moving a mass at right angles to the vertical) and will be,

$$\begin{array}{lcl} \text{From } Q \text{ to } P, & dg & \parallel \text{ From } Q' \text{ to } P' & d'(g + \Delta g) \\ \text{From } P \text{ to } P' & 0 & \parallel \text{ From } Q \text{ to } Q' & 0 \end{array}$$

$$dg = d'(g + \Delta g)$$

If PQ and $P'Q'$ are straight lines, $d = d'$ and $\Delta g = 0$, the value of gravity is the same along PQ and $P'Q'$, and there is no gradient between P and P' and between Q and Q' .

If PQ and $P'Q'$ are curved, $d \geq d'$ and then $(g + \Delta g) \geq g$, Δg has a definite value, and there is a gradient from P' and Q' to P and Q .

The proof of the second theorem is as follows: In Fig. 6 let: aa' be the level surface through the center of gravity of an Eötvös torsion balance system.

$abca'$ and $aeda'$ be two vertical unit squares (1 cm. on a side),

V be the line of the vertical,

g the value of gravity at a ,

$gr(g)$ the gradient from a to a' ; then the $(g + gr(g))$ will be the value of gravity at a'

The vertical will be perpendicular to aa' but on account of its curvature will make an angle, α , with bc and ed .

The work done, if a unit of mass is moved from d to a , is the same whether the path of the movement is via $da'a$ or dea and will be:

| | |
|--------------------------------|-----------------------------------|
| From d to a' , $g + gr(g)$ | From e to a , g |
| From a' to a , 0 | From d to e , $g \cos \alpha$ |
| (adding) $g + gr(g) =$ | $g + g \cos \alpha$ |
| and $gr(g) =$ | $g \cos \alpha$ |

$(g \cos \alpha)$ is the small horizontal force, which is set up because, above or below the level of the center of gravity, the vertical is not perpendicular to the horizontal and gravity therefore has the small horizontal component $(g \cos \alpha)$. This small horizontal force acting on one of the weights

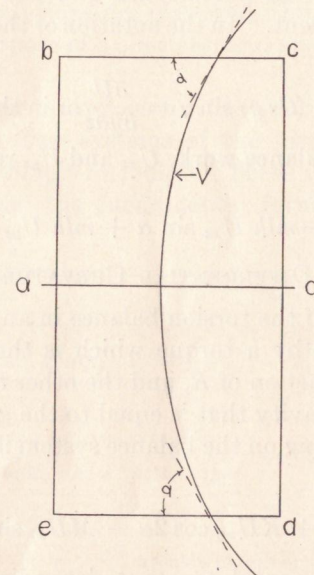


FIG. 6.—VERTICAL SECTION THROUGH AN EÖTVÖS TORSION BALANCE SYSTEM REPRESENTING THE CURVATURE OF THE VERTICAL.

of a torsion balance will tend to rotate the balance. As the situation in $abca'$ is symmetrically reversed from that in $aeda'$, the force acting on the upper weight of an Eötvös torsion balance is equal to that acting on the lower weight, and the same in rotational effect, but opposite in absolute direction. As α is very close to 90° , $\cos h \cdot \alpha = h \cos \alpha$ and as h is the vertical distance between the weights, that small horizontal force acting on each weight $= \frac{h}{2} g \cos \alpha$. As $g \cos \alpha = gr(g)$, the torsion balance is able to measure the gravity gradient.

The actual magnitude of the force F acting on the weights of the balance in any particular position is proportional to the maximum gradient $(Gr g)$ and to the sine of the angular difference between the azimuths

of the balance bar and of the gradient. If the azimuth of the balance bar referred to the direction of the gradient as the axis of reference is $+\beta$, then

$$F = -mlh(Gr g) \sin \beta. \quad (10)$$

If the axis of reference is transferred to north and, if in the derivation of the transformation equations, the gradient is considered to be in the northeast quadrant, and if α = the new azimuth of the balance and if γ = the azimuth of the gradient:

$$\beta = \alpha - \gamma \text{ and } F = -mlh[(Gr g) (\sin \alpha \cos \gamma - \cos \alpha \sin \gamma)] \quad (11)$$

The quantities $(Gr g) \cos \gamma$ and $(Gr g) \sin \gamma$ are constants for any particular gradient and represent the north-south and the east-west components respectively of the gradient. In the notation of the calculus of the theory of gravitation

$$(Gr g) \cos \gamma = \frac{\partial U}{\partial x \partial z} \text{ and } (Gr g) \sin \gamma = \frac{\partial U}{\partial y \partial z} \text{ or in the abbreviated notation}$$

in use in the torsion balance work, U_{xz} and U_{yz} respectively. Equation (11) then becomes:

$$F = -mlh U_{xz} \sin \alpha + mlh U_{yz} \cos \alpha \quad (12)$$

CALCULATION OF THE DIFFERENTIAL CURVATURE AND THE GRADIENT

The balance beam of the torsion balance in any particular position of the balance is affected by a torque which is the vectorial sum of two torques, the one the function of R , and the other the function of the horizontal component of gravity that is equal to the gradient. The value of the total torque, F , acting on the balance system is given by the following equation:

$$N = \frac{K}{2} U_{\Delta} \sin 2\alpha + K U_{xy} \cos 2\alpha - M U_{xz} \sin \alpha + M U_{yz} \cos \alpha \quad (13)$$

Where: K is an instrumental constant depending on the mass of the weights, the square of the half length of the balance bar, and on the reciprocal of the torsion constant of the torsion wire; and M is an instrumental constant depending on the mass of the weights, the half length of the balance bar, the vertical distance between the weights, and the reciprocal of the torsion constant of the torsion wire; and τ is the torsion constant of the torsion wire in scale divisions of the instrument; and N = angular rotation of the balance in scale divisions; and $F = N\tau$

The torque F causes an angular rotation of the balance bar until counterbalanced by the resistance of the torsion wire. The angle through which the beam turns is determined by reading the deflection in a mirror on the stem of the balance of a fixed point without the rotating system.

If four values for N are determined by observation, the equations can then be solved for the values of the four unknowns, U_{xz} , U_{yz} , U_{xy} , and U_{Δ} . From the nature of the torsion balance, however, the zero point of

the instrument enters the equation as a fifth unknown in the case of the obsolete, single-balance type of instrument and then at least five determinations of N are necessary. As the double-balance modern type of instrument consists of two complete independent balance systems mounted closely side by side but with a difference of orientation of 180° , the zero point of each system enters the equation as an unknown; and therefore with the modern instrument at least six independent determinations of N are necessary. As two determinations of N , one by each system in the instrument, are made in each position, the observation of the modern instrument in three positions is sufficient mathematically for the determination of all six variables, but as a matter of fact, at least four and usually five positions are observed, the last two a repetition of the first two for the purpose of a check on and an increase in the accuracy of the observations.

The solution of those equations for the gradient, the curvature, and the zero points of the two systems of the instrument, in practice, is replaced by the use of simple formulas. These very commonly are used in the form of a table, the fundamental form of which is somewhat as follows:

| $n_0' = \frac{1}{2}(n_1' + n_2' + n_3') =$ | | | | $n_0'' = \frac{1}{2}(n_1'' + n_2'' + n_3'') =$ | | |
|--|---------------------|--------------------------|-----------------------|---|---|---|
| I $(R_2' - R_3')$ | II $R_2' + R_3'$ | III $R_2'' - R_3''$ | IV $R_2'' + R_3''$ | $R_1' = (n_1' - n_0') =$ $R_1'' = (n_1'' - n_0'') =$ | $R_2' = (n_2' - n_0') =$ $R_2'' = (n_2'' - n_0'') =$ | $R_3' = (n_3' - n_0') =$ $R_3'' = (n_3'' - n_0'') =$ |
| | | * | | I - III = | II - IV = | |
| V = $2R_2' + R_1' =$ | | VI = $2R_3'' + R_1'' =$ | | VI - V = | $R_1'' - R_1' =$ | |
| VII = $2R_2' + R_1' =$ | | VIII = $2R_2'' + R_1' =$ | | VIII - VII = | | |
| $U_{xz} = B \cdot IX =$ | | $U_{yz} = A \cdot X =$ | | IX = Mean = | X = Mean | |
| Gr(g) = + $\sqrt{U_{xz}^2 + U_{yz}^2} =$ | | | | Azimuth Gr(g) = $\arctan \frac{U_{yz}}{U_{xz}}$ | | |

where n'_0 and n''_0 are the zero points respectively of the two systems of the instrument; n'_1 , n'_2 , n'_3 , and n''_1 , n''_2 , n''_3 are the respective scale reading of the two systems read in the 0° , 120° , 240° azimuths measured clockwise from north; and A and B are instrumental constants. The Roman numerals indicate the sequence of operations.

The curvature values are calculated by similar extension of this table.

This and the similar tables lead through a simple, indicated routine to the north-south and east-west component of the gradient, and to U_{Δ} and U_{xy} . The calculations involved are the simplest operations of addition, subtraction and multiplication. The last step of obtaining the total gradient and its azimuth, or R and its azimuth, is then performed easily by entering simple tables or graphs respectively with U_{xz} and U_{yz} or U_{Δ} and U_{xy} .

USE OF THE EÖTVÖS BALANCE IN GEOLOGY

The values recorded by the instrument respectively for the gradient and the differential curvature at any station are the vectorial sums of the effects of all the irregularities of distribution of mass around the instrument that are of sufficient size and proximity to the instrument sensibly to affect it. Those effects, genetically, are of three types: the topographic, the planetary, and the geologic.

TOPOGRAPHIC ANOMALIES

The topographic anomalies are caused by the irregularities in the distribution of mass due to the hills and valleys, mounds and depressions

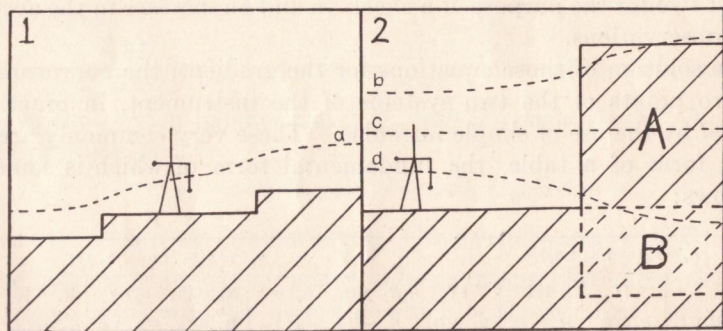


FIG. 7.—DIAGRAMMATIC REPRESENTATION OF THE VARIATION OF GRAVITY IN THE VICINITY OF AN EÖTVÖS TORSION BALANCE.

1. Due to a depression of the surface on the left and a slight rise of the surface on the right of the balance.
 2. Due to a hill or cliff, A, rising above the level of the instrument and a depression, B, which would produce identically the same gradient at the instrument as A.
- a, b, c, d are profiles of the variation of the value of gravity.

surrounding the instrument, rising above or sinking below the horizontal plane through the foot of the instrument. If the terrane in which the instrument is set up is a flat horizontal plain, there is no gradient or curvature effect; but if there is a depression below that plain, gravity is less than normal above the depression, a gradient is set up away from the depression and the level surfaces are warped down, concavely, above the depression (Fig. 7-1). If the surface rises above the plain, gravity is greater than normal above the elevation, a gradient is set up toward the elevation, and the level surfaces are warped up, convexly, above it. But if a cliff or hill rises more than a meter above the center of gravity of the instrument, the mass of the hill or cliff above the level of the center of gravity of the instrument exerts an attraction upward and therefore tends slightly to counteract gravity, to set up a gradient away from the cliff or hill, and to warp the level surfaces down into the hill or cliff (Fig. 7-2).

Corrections for the respective topographic anomalies of the gradient and differential curvature are applied to the values observed at each station. The level of the ground around the instrument is determined with an alidade or transit at a definite net of points, most commonly at distances of 1.5, 3, 5, 10, 20, 50, 100, and very rarely 200, 500, and 1000 m. from the instrument on azimuths of 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°. The density of the soil is measured at two or three points near the instrument in the more accurate work, but for most purposes sufficient accuracy is obtained by using a mean density for the area, and furthermore, in many places, indeterminable irregularities in the zone of weathering make unnecessary great refinement in the determination of the soil density. The calculation of the respective topographic effects of gradient and differential curvature is made in practice by one of several simple rule of thumb formulas which have been derived and cast in simple tabular form. The elevations obtained in the levelling are entered in the proper spaces in the table; and a few operations of simple addition and subtraction and of multiplication by coefficients give the total anomaly due to the topography. That effect is then subtracted algebraically from the respective observed value.

The effect of a small mass on the gradient observed varies both with the horizontal and vertical position of the mass in reference to the instrument. There is no effect on the gradient, if a small mass is on a level with or perpendicularly below the instrument; the effect is at a maximum when the mass is 40° to 60° above or below the horizontal plane through the center of gravity of the instrument. The effect of irregularities of mass within 2° of that horizontal plane are negligible except in very refined work and within $\pm 5^\circ$ in most work. The effect also varies inversely as the cube of the distance of the mass from the instrument. In regions of low to moderate relief, in most torsion balance work it is possible to choose station sites so that all large irregularities of topography are within $\pm 5^\circ$ of the horizontal plane through the center of gravity of the instrument, are moderately distant from the instrument, and therefore can be neglected. If the instrument is set up on an extensive hill sloping uniformly 1° the effect of the slope on the gradient amounts to 14 Eötvös units, and for slightly steeper slopes increases directly as the number of degrees of slope. Unless an impracticable number of drill holes are sunk for samples for the determination of the density of the soil and the subsoil, the value used may be in error to the amount of 10 per cent. If a surficial zone of weathering is present, the surface of the unweathered rock acts like another topographic surface, but as it conforms only very roughly to the surface of the ground, its effect on the gradient and the differential curvature is only very roughly proportional to the effect of the actual surface of the ground, and in practice can be determined only by an impracticable number of core holes through the zone of weathering. As

the gradients due to geologic structure commonly are of the order of 7 to 25 E, it is advisable to choose station sites where the inclination of the ground is less than 1° and preferably less than 0.5° . In regions of low or moderate relief, station sites usually can be chosen to have slopes of less than 1° , except where stations have to be close together. In regions of rugged or mountainous relief, usually it is difficult to pick station sites where the slope of the ground is less than 3° and where hills do not arise more than 5° above or valleys extend more than 5° below the horizontal plane of the center of gravity of the instrument and where those hills and valleys are not relatively near the instrument. If a reasonably accurate topographic map is available, a value for the effect of the topography beyond 1000 m. can be obtained by rather laborious and tedious calculation, but it and the value for the effects within the 1000-m. zone of leveling may easily be very much in error. In rugged country, therefore, it is not practicable to use the torsion balance for surveys of gravity gradients of structure unless the gradients caused by such structures are very large. The terrane correction formulas now in common use for the zone within 100 m. are not valid for elevations of more than 0.5 m. above the level of the base of the instrument.

The effect of a small mass on the differential curvature value observed varies with the horizontal and vertical position of the mass in reference to the instrument. The effect is proportional to the effect of the attraction of the mass in deflecting the vertical, and is at a maximum when the small mass is in the horizontal plane through the center of gravity of the instrument and at a minimum when the small mass is vertically above or below the center of gravity of the instrument. The effect varies inversely as the cube of the distance of the mass from the instrument. As the effect is very nearly at its maximum value within 10° of the horizontal plane of the center of gravity of the instrument, it is difficult even in regions of moderate relief to choose stations so that the effect of the topography will not be serious. With increasing ruggedness of topography, the difficulty of eliminating the effects of the topography increases very much more rapidly with the differential curvature than with the gradient, and the differential curvature usually is considerably more erratic than the gradient.

Planetary Effects on Gradient and Differential Curvature

The planetary effects are due to the fact that the earth is a rotating spheroid flattened along the polar axes, instead of a perfect sphere at rest. On account of the flattening and the rotation of the earth, the value of gravity at sea level increases from the equator to the poles and therefore there is a northward gradient in the northern hemisphere and a southward gradient in the southern hemisphere. As at any latitude, there is

no variation of gravity eastward or westward, there is no east or west component comparable to that northward and southward gradient. The level surfaces likewise are distorted by the flattening and rotation of the earth. If the earth were a homogeneous sphere at rest, all level surfaces would be spheres and the radius of curvature of the level surfaces would be the same in all directions, but on account of the flattening and rotation of the earth there is a change of the radius of curvature in meridional planes, and U_Δ therefore comes to have a value; but as our axes of reference, N-S and E-W are parallel to the major and minor axes of the level surfaces, $U_{xy} = 0$. The values of the "normal" northward gradient and the "normal" differential curvature, as the planetary effects are called in torsion balance work, vary with latitude and can be calculated mathematically when appropriate assumptions are made for the form and rotation of the earth. A table of these values usually is furnished to each observer and he has only to enter his table with the approximate latitude of his station in order to get values respectively for the normal northward gradient and the normal curvature. Those values then are subtracted from the respective observed values.

The normal effects and the terrane effects normally are subtracted in the calculation of the observed values at each station and in most torsion balance work, the term "observed gradient (or differential curvature)" is used to refer to the gradient (or differential curvature) produced by the geologic anomalies.

Geologic Anomalies

The geologic anomalies are those produced by the irregular distribution of mass in the upper few miles of the earth's crust and range from effects produced by irregularities of mass of subcontinental size to those produced by small boulders close to the instrument. These anomalies may be divided somewhat arbitrarily, on the basis of the size of the geologic structure producing them, into five orders of magnitude: first order, those due to masses of the size of large mountains or large geosynclines, such as the West Texas Permian basin; second order, those due to masses of the size of small mountain ranges such as the Amarillo buried granite ridges; third order, those due to masses of the size of the average anticline; fourth order, those due to minor geologic structure; fifth order, those due to small surficial irregularities of mass, such as glacial boulders buried in the subsoil rather close to the instrument.

The surficial irregularities of mass are due to a wide variety of geologic causes. In glaciated regions, a buried glacial boulder, the rapid variation possible in fluvio-glacial deposits, the contrast between till and fluvio-glacial material, or the irregularities in the surface of the bed rock buried under a thin mantle of glacial deposits, may cause abrupt changes in

specific gravity within a short distance of the instrument and thereby cause very abnormal gradient and differential curvature values to be registered by the torsion balance. Where a river valley has been filled with alluvium, a very large gradient away from the side of the valley, and very abnormal curvature values, may be set up over a steeply inclined contact of alluvium and bed rock at the edge of the valley. If the alluvium and bed rock have sufficiently contrasting specific gravities, the whole bed-rock contour of the valley floor will sensibly affect the gradient and curvature values at the surface and in such cases the torsion balance can be used to map the position and conformation of a buried valley. In some formations, irregularities of cementation, heterogeneity in the character of the formation, intercalated beds of limestone or dense sandstone, may cause a sharp change in density close to the instrument and give rise to abnormal gradient and curvature values. In the area of the Wilcox formation of Texas for example, both the gradient and curvature not uncommonly are distinctly erratic. In the Gulf Coast, a sandy pimple mound rising through a clayey soil may cause a sufficiently abrupt change in density to give abnormal gradient and curvature values. A filled-in cellar hole or other artificial excavation or small faults extending to the surface may produce sufficiently large surficial irregularities of mass to produce similar effects.

The abnormal gradient and differential curvature values caused by the surficial irregularities of mass are limited to a very small area and in most cases will not be registered if the station site is moved a few hundred feet. An abnormal gradient or differential curvature value very often will be suspected by an experienced interpreter of torsion balance results. As the surficial irregularities of mass are hidden beneath the surface and could be detected and mapped only by an impracticable number of core holes, it is impossible to calculate their effects as the effects of irregularities of topography can be calculated. The effects of the buried bed-rock-alluvium contact at the edge of a valley and the outcropping beds of limestone, hard sandstone or other dense rocks can be avoided in many places if the observer watches the geology and chooses the station site with care. The technique of minimizing those effects, where they can not be avoided, varies with the situation. On reconnaissance or on profile surveys, two, three, or in some cases four or five stations may be occupied a few hundred feet from and around a common point and the mean or median value used as the value of the gradient (or differential curvature) at that point. Where a considerable net of stations would be occupied in any case, it may be preferable to double the number of stations and then make a least square adjustment of the values.

The geologic anomalies of the second, and third and fourth order, give the geologist clues to geologic structure. But the gradients and differential curvature observed at any point will be the sum of the

sensible effects of all those orders. An anomaly is the same order of magnitude as the structure producing it. If a torsion balance survey is extensive enough and there is a sufficient net of stations, a careful analysis of the results may allow a separation of the anomalies of different orders (See discussion of the Fox oil field, page 41).

GEOLOGIC INTERPRETATION OF TORSION BALANCE RESULTS

The use of torsion balance results as a guide to geologic structure depends on two assumptions:

1. That the form, depth, and relative density of a mass causing a gradient and curvature anomaly can be determined from the distribution of the gradient and curvature values in the anomaly observed at the surface.

2. That there is a direct connection between geologic structure and the anomalous distribution of mass.

The accuracy of these assumptions varies with the situation and ranges from a high order to a low order. The accuracy of the interpretation of geologic structure from torsion balance results depends on the extent to which those assumptions hold true—and on the skill of the interpreter.

Any particular body has a characteristic gradient and curvature pattern which depends on the form and size of the body and its depth below the surface. The amplitude of the gradient and the differential curvature values depends on the difference between the respective densities of the body and the surrounding medium. The gradient and curvature profiles for four common simple types of geologic oil structure are shown in Fig. 8. The structures (with the exception of that in Fig. 8e) are supposed to be infinitely long with the respective transverse cross-sections shown in the figures. The lower portion of each figure gives the structural cross-section drawn with vertical and horizontal scales equal. The upper profile gives the curve of the variation of the value of R , the differential curvature, above the structure; and the lower profile gives the variation of the gradient above the structure. On the differential curvature profile, each point on the curve above the zero line would be represented on a map by an R line of equal magnitude parallel to the line of the section, and each point on the curve below the zero line would be represented by an R line at right angles to the line of the section. On the gradient profile, each point on the curve above the zero line would be represented on a map by a gradient arrow flying to the right and each point on the curve below the zero line would be represented by an arrow flying to the left.

A vertical fault cutting a shale section of 3000 to 3500 ft. thick, overlying a very thick massive limestone section, is represented in Fig. 8a. The fault has a throw of 300 ft. at the surface and at 1000 ft.; a throw of

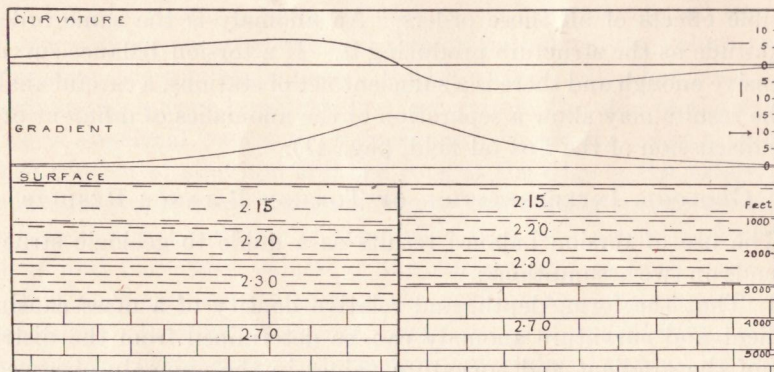


FIG. 8.—COMMON SIMPLE TYPES OF GEOLOGIC STRUCTURE AND THEIR GRADIENT AND DIFFERENTIAL CURVATURE PROFILES.

a. Vertical fault.

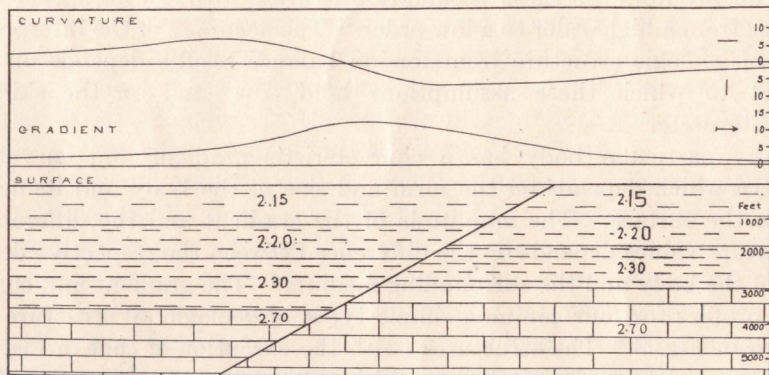


FIG. 8.—b. 30° fault.

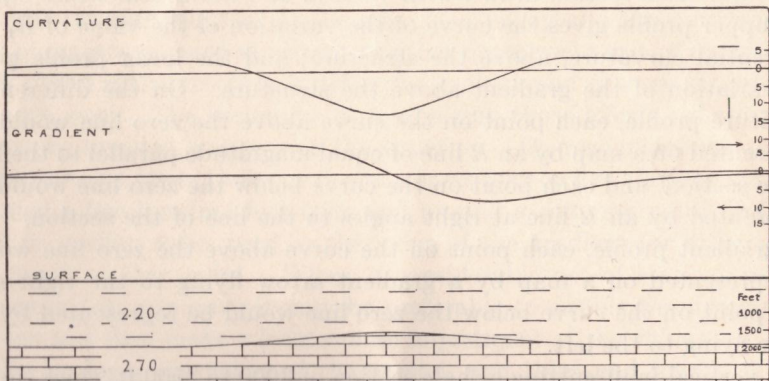


FIG. 8.—c. Symmetrical infinite (anticlinal) ridge.

400 ft. at 2000 ft., and a throw of 500 ft. at 3000 ft. The shale section is supposed to be split into three formations with specific gravities, increasing downward, of 2.15, 2.20, and 2.30 respectively. The specific gravity of the limestone was assumed to be 2.70. Both the gradient and differential curvature are symmetrically disposed in reference to the fault plane: the differential curvature is zero and the gradient is at a

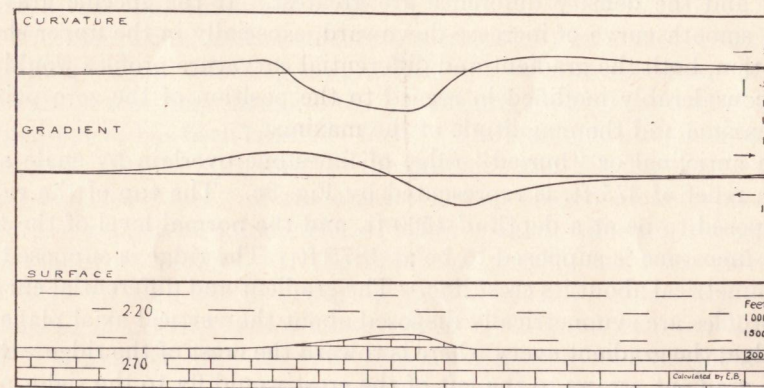


FIG. 8.—d. Asymmetrical infinite (anticlinal) ridge.

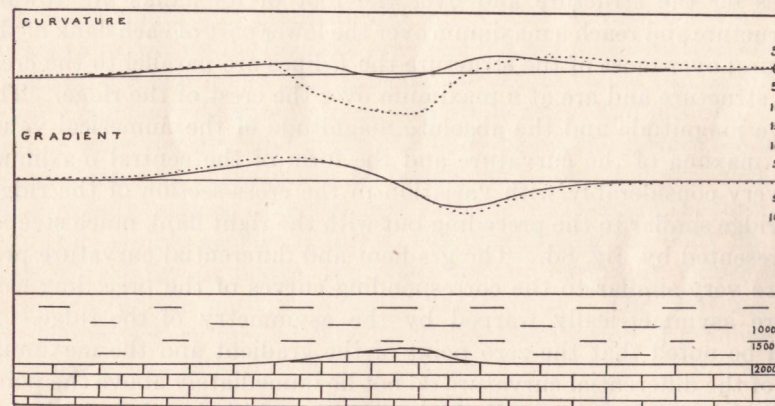


FIG. 8.—e. Ridge of the same cross-section but with a length 2 times the depth at each level. The dotted gradient and curvature profiles represent the profiles for the infinite case.

maximum over the fault; on the left of the fault plane the R lines are perpendicular to the fault line, and on the right are parallel to it; the gradient arrows all fly toward the right but increase to a maximum over the fault and then decrease symmetrically.

A fault with a dip of 30° cutting the same stratigraphic section as in the preceding case is represented by Fig. 8b. The gradient and differ-

ential curvature profiles are very similar to those of the preceding case but are not symmetrical, the gradient maximum and the numerical maxima of the curvature are slightly less than the corresponding maxima of the preceding case, and the zero of the differential curvature and the maximum of the gradient do not lie above the surface trace of the fault but over the upper part of the fault in the limestone basement, where the throw and the density difference are greatest. If the specific gravity had a smooth curve of increase downward, especially in the upper shale formation, both the gradient and differential curvature profiles would be very considerably modified in regard to the position of the zero points and maxima and the magnitude of the maxima.

An anticlinal or "buried" ridge of limestone overlain by shale and with a relief of 375 ft. is represented by Fig. 8c. The top of the ridge is supposed to be at a depth of 1500 ft. and the normal level of the top of the limestone is supposed to be at 1875 ft. The ridge is supposed to be symmetrical about its crest line. The gradient and differential curvature profiles are symmetrically disposed about the vertical axial plane of the ridge; the gradient every where is toward the crest of the ridge—that is, the gradient arrows to the left of the crest would fly to the right, and on the right of the crest, they would fly to the left; the gradient is zero above the crest of the ridge; and the maxima are over the flanks. The *R* lines off the structure and over the foot of its flanks are toward the structure and reach a maximum over the lower part of each flank of the structure; over most of the structure the *R* lines are parallel to the crest of the structure and are at a maximum over the crest of the ridge. The relative magnitude and the absolute magnitude of the numerical value of the maxima of the curvature and the form of the central maximum vary very considerably with variation in the cross-section of the ridge.

A ridge similar to the preceding but with the right flank much steeper is represented by Fig. 8d. The gradient and differential curvature profiles are very similar to the corresponding curves of the preceding case but are asymmetrically warped by the asymmetry of the ridge. It should be noted that the zero point of the gradient and the maximum value of the differential curvature do not lie immediately above the point of the ridge that is structurally highest but are shifted slightly to the left down the gentler slope of the ridge. The zero point where the gradient changes direction marks the point of maximum gravity and it should be noticed that in asymmetric structures the center of the anomalous gravity high due to the structure is not exactly over the highest point of the structure; the amount of the shift of the gravity high down the gentler slope of the structure is proportional to the difference in the slope of the two flanks. In unknown structures, the shift of the gravity high due to the asymmetry of the structure can be recognized if a moderately detailed transverse profile has been run but its magnitude can not be determined

unless quantitative calculations are made. If only a scattered net of stations is available, the presence of the asymmetry may not be recognized easily, and a location made from the torsion balance data may be off just enough to miss the crest. As the gradient and differential curvature profiles of Fig. 8e show, the gradient profile is slightly modified and the differential curvature profile very considerably modified if the ridge is finite rather than infinite in length.

The dimensions of the structure sections of Fig. 8 may be taken as anything without affecting the gradient and curvature profiles. In order that the reader may visualize the dimensions readily, the unit of spatial measurement has been labelled "feet." The unit of spatial measurement does not enter the formulas for the gradient and the differential curvature, and the "feet" of the structural sections could be replaced by inches, centimeters, kilometers, or miles without affecting the gradient and curvature profiles. If the profile of gravity has been plotted, it would be affected by the change of unit of spatial measurement, as the difference of gravity between two points is the product of the gradient and the distance, and the unit of spatial measurement is involved in the distance.

Relation of Subsurface Bodies to Gradient and Differential Curvature

Any given simple body at a given depth below the surface produces a unique distribution of gradient and differential curvature values at the surface; and, within the limits of practical but not mathematical accuracy any given distribution of gradients and curvature values can be produced by only one body definite in form, relative density⁵ and position. If the form, relative density, and depth of a body below the surface are known, the gradients and curvature at all points can be obtained by calculations which rapidly increase in tediousness with the deviation of the form of the body from simple geometric forms, but by splitting the body up into a series of rectangular plates, the gradients and curvature values can be calculated for the most irregularly shaped body. The converse holds true only to a very limited extent. If the distribution of gradients and curvature values caused by a body are completely known, it is practically impossible accurately to determine the shape, position, and density of the body. If the gradient profile produced by an infinite rectangular block is known fairly completely, it is possible to calculate the depth to the top and the bottom of the block, the position and the relative density of the block with a good degree of approximation. For simple tabular bodies with cross-sections similar to those of Fig. 8, fair approximations can be obtained for the shape, position and relative density of the body by only moderately tedious calculations based in part on methods of the

⁵ The relative density of a mass = the density of the mass minus the density of the surrounding medium.

trial and error, and with only slightly greater tediousness for a block of finite length compared to that for a similar block of infinite length.

If a body producing a given distribution of gradients and differential curvature values is not homogeneous with a simple geometrical form, is extremely irregular in form, or is a composite of several simple homogeneous bodies of differing densities, or if it has considerable variation of density, it is much more tedious and in some cases impossible in practice to calculate back and obtain the data of the body.

If a geologic body is homogeneous in density, if it has a relatively simple geometrical form, if the surrounding country rock is homogeneous in density and if a sufficiently close net of stations has been occupied, it is possible to calculate back from the observed distribution of gradient and curvature values and obtain approximately the form, position, dimensions and density of the body. If a geologic body is known to be composed of two rather simple homogeneous bodies, the form, position, dimensions, and densities of the two bodies in some cases can be determined from the calculations.

If a geologic body is irregular in form and density, it is impossible to calculate its form, position, dimensions and density, but it is possible to calculate an imaginary simple homogeneous body that most nearly approximates the given irregular body. The significance of the imaginary body depends on the closeness of its approximation to the geologic body; and that is difficult of evaluation unless the geologic probabilities of the situation are known and are very limited.

Accuracy of Quantitative Calculations

All the results of such quantitative calculations, even of the most certain, are subject to varying degrees of uncertainty. Bodies differing considerably in form, position, and density may give respective distributions of gradient and curvature values that differ only by a very few Eötvös units. Except where most painstaking precautions are taken in exceptionally favorable areas, the observed gradient and curvature values may be in error by one, two, three, or even more Eötvös units on account of surficial irregularities of density and of inexact correction for the effect of topography. Furthermore, it is usually impracticable to put stations close enough together to get an exact picture of the distribution of gradient and differential curvature values. Although the general plan of the distribution of gradient and differential curvature values will be known, the detailed plan will be sufficiently indefinite so that a number of detailed profiles of equal probability may be chosen. Corresponding to each profile there will be a definite body, which will differ slightly in form, shape, dimensions, position and density from the bodies corresponding to the other of those plans. Any one of these bodies will have as great a

probability, mathematically, as the others, but some of them may not be geologically possible or as probable as the others.

A common result of such calculations is to obtain a suite of bodies that correspond to a suite of density differences, which have a distinct family resemblance to each other but differ essentially only in depth and vertical relief. The suite of bodies and corresponding suite of densities in most places have finite limits and although the calculations do not lead to a definite determination of the structure, they may define it within certain limits. In the calculation of the form of the Ordovician-metamorphic basement along a profile that had two highs, it was impossible to make any reasonable assumptions that would allow the depth to the top of the Ordovician at the crest of the first high to be greater than 1550 ft. below the surface or on the second high to be closer than 2300 ft. to the surface. A test subsequently drilled below the crest of the first high encountered the Ordovician at a depth of 1453 ft. below the surface; and a test previously drilled below the crest of the second high had found the top of the Ordovician at a depth of 2665 ft. below the surface. Calculations of that type in general should give the position of the crest of the structure with much more definiteness than the other data regarding it. If the calculations are based on a survey in which the stations were not spaced sufficiently close together completely to define the respective gradient and curvature profiles, additional uncertainty is added to the results of the calculations to approximately the same extent as the assumptions have to be made in regard to the variation of the gradient and the differential curvature between the stations at which they have been observed.

Determining Geologic Structure with Torsion Balance

As the torsion balance measures only the effect of irregular distribution of mass, the use of torsion balance results to determine geologic structure, of necessity, is dependent on the assumption of a concordance between the geologic structure and the distribution of mass. The degree of the concordance depends on the type of the geologic structure and the particular situation. Concordance relatively complete for practical purposes holds directly in many cases of salt domes, volcanic plugs, laccoliths, dikes, batholiths, intruded into homogeneous country rock, and in the case of a few types of ore deposits in homogeneous country rock. The concordance becomes increasingly incomplete as the density of the country rock becomes heterogeneous.

Indirect partial concordance prevails in most of the anticlinal, domed, and fault structures in which the oil geologist is interested. The parallelism between the structure and the distribution of mass is due to the fact that normally there is little variation in the same bed, although there may be vertical variation between beds. Deformation of the beds by

folding or faulting therefore involves concomitant deformation of the distribution of density. There is a general tendency for increasing consolidation of sediments with increasing geologic age—except in the case of limestones, anhydrite, and salt—and therefore in general for increasing density with increasing depth. The igneous and metamorphic rocks, and the early Paleozoic sediments forming the buried mountain ranges of the Mid-Continent area in general have a higher density than the overlying sediments. A general tendency therefore prevails for the association of gravity highs with structural highs; but the reverse association is possible, and will be given by a massive salt series lying not too deep below the surface in otherwise homogeneous sediments or by a massive limestone at the surface underlain by a massive sand-shale section.

The intensity of the folding in oil structures and therefore of the deformation of the uniform distribution of density tends to increase downward. In the case of the buried-ridge type of structure, there is a tendency for the influence of the basement complex to be predominant in the gradient and differential curvature values, yet in all cases the deformation of the uniform distribution of density tends to parallel the structural deformation, and the intensity of the first tends to be of the same relative magnitude as the second.

Types of Geologic Interpretation of Torsion Balance Results

The geologic interpretation of torsion balance results may be of two types: quantitative and qualitative. If the geologic possibilities and probabilities are known well enough to show that a body possesses a high degree of direct concordance between itself and the distribution of density, and has a rather simple geometrical form it is possible to calculate backward from the gradient and curvature values and determine the form, dimensions, and density of the body, and thereby make a "quantitative" or approximately quantitative interpretation of the torsion balance results. A geophysical geologist who is familiar with the mathematical theory of interpretation, who has had considerable practicable experience and who has a fair knowledge of the geology of the area, by studying the distribution of the gradients and curvature values, the maximum and minimum values, the position of the reversals and changes in direction of the gradients and curvature values, and the rate and character of the variation of the gradient and curvature values and directions, will find in most (but not all) torsion balance surveys a fair general picture of the structural conditions of the area surveyed. He will see indications of faulting or folding; of the position of the major faults and folds; of the steepness of the dip of a fault and of the approximate size of a fold; of whether the flanks are steep or gentle; of the presence of any abnormally massive bed or formation of abnormally high or low density, and of its approximate depth.

This determination of the general and approximate form, size, position and density of the anomalous mass is what the writer calls "qualitative interpretation." In simple structural situations, a competent interpreter can make a good qualitative interpretation of the situation by simple inspection of the results; but where the structural situation is complicated or the gradients and curvature values small, refined analysis may be necessary. A map showing the contours of equal gravity (isogams) which will resemble a structure contour map, and which may be used in many respects as a structure contour map, can be constructed if the net of stations covers the area sufficiently (Figs. 12 and 13). There is, however, no constant factor between the isogams and the structure contours, even in a single structure, and the point of maximum intensity of the anomalous gravity does not lie necessarily above the crest of the geologic structure. If the gradient and differential curvature produced by larger scale structure obscure those of smaller structure in which the geologist is more particularly interested, in many cases the gradient and differential curvature anomalies of the latter can be separated out by the proper analysis (Figs. 13a, b, c).

Although satisfactory in a very great many cases, the "primary net" method of least-square adjustment of the gravity difference between stations used by Eötvös is not quite accurate enough in the more refined analysis and it is necessary to make a simultaneous adjustment of all stations. As such a simultaneous adjustment involves the solution of as many linear equations as there are stations, the calculations become extremely tedious, unless approximate formulas are available to the calculator. Anomalies obtained by such refined analyses are somewhat treacherous and should be used only by an experienced interpreter who is aware of the dangers in their use.

RESULTS OF TORSION BALANCE WORK IN THE UNITED STATES

Three definitely, and possibly five, salt domes have been discovered by the torsion balance in the Gulf Coast salt dome district of southeast Texas and southwest Louisiana within the three years 1924–26 in contrast to the discovery of only five new domes by geology in the preceding fifteen years.

Discovery of the Nash Salt Dome

The first of the salt domes—the first oil structure—discovered in this country by geophysical instruments was the Nash salt dome, discovered in the early Spring of 1924 by the Rycade Oil Corp. This had been considered a faintly suspicious prospect; it was rated by the writer as having about five chances out of one hundred of being a salt dome, but apparently rated not so favorably by the geological departments of some other companies. It was blocked by the Rycade Oil Corp. and the

reconnaissance torsion balance survey, of which the results are shown in Fig. 9, was made; this indicated the presence of a salt dome with a probability of better than ninety-nine chances out of one hundred. The dashed lines in the figure show the limits of the dome as they were interpreted by the writer from the torsion balance results entirely in advance of drilling;

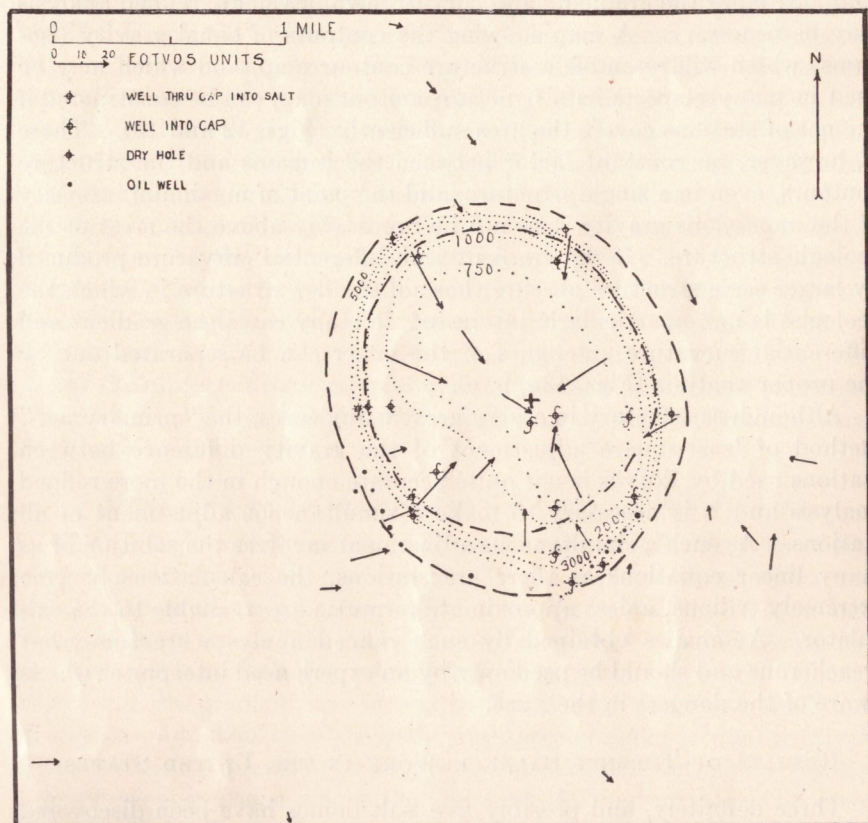


FIG. 9.—ORIGINAL RECONNAISSANCE TORSION BALANCE MAP OF THE NASH PROSPECT AND THE STRUCTURE CONTOUR MAP AS SUBSEQUENTLY DETERMINED BY DRILLING.

The structure contours are on the top of the cap or, in the absence of the cap, on the top of the salt. (Survey by the Rycade Oil Corp. under the direction of the writer. Published by permission of E. L. DeGolyer, President, Rycade Oil Corp.)

the inner dashed line represented the edge of the salt table at a depth estimated to be somewhere between 500 and 900 ft. and the outer dashed line represented the estimated position of the salt at 4000 to 5000 ft. The prediction of the position of the center of the dome is shown by the heavy cross in the figure. The first well, drilled just south of the interpreted center of the dome, demonstrated the presence of the dome. The second well, located on the center of the east edge of the dome, showed that the edge of the dome had been placed slightly too far out.

Our interpretation at that time was based on an empirical study of many of the known salt domes, and we had not yet made a study of the mathematical phases of interpretation and of the types of gradient profiles produced by masses of different shapes at varying depths. From our empirical studies we recognized that on any one dome the position of the maximum gradients and the rapid decrease of the gradients outward from the dome to zero seemed to be in a tolerably constant relation to the edge of the dome but that from dome to dome there was considerable variation in the relation; and in outlining the edge of the dome from the torsion balance results, we expected that the first well drilled on the edge of the dome would cause a revision of our interpretation of the position of the edge, a revision that should bring the outlined position of the edge of the dome into fair agreement with the actual for the whole of the rest of the dome. A radial contraction of 500 to 600 ft. in the position of the edge of the dome, as outlined in Fig. 9, brings the predicted position of the edge into very fair agreement with its actual position, except on the south. If consideration is taken of the facts that the Gulf Coast salt domes range in diameter size from less than a quarter of a mile to over three miles, and that they may be circular to slightly elliptical in plan, the delimitation of the edge of the dome with an accuracy of better than 1000 ft. was a very great advance on all previous methods of delimiting a dome not indicated by a well defined mound. As the result of later experience, we would not attempt now to make an accurate interpretation of the position of the edge of the dome from so sketchy a survey as that original reconnaissance survey at Nash.

Other Domes Indicated by Torsion Balance

The other four salt domes discovered by the torsion balance are the Clemens, Allen, Long Point, and Fannett domes. The Clemens dome was discovered at a locality where the presence of a dome was not suspected. The Allen dome, like the Nash dome, was at a locality where there were some very faint indications of the possible presence of a salt dome. Both domes were discovered by the Roxana Petroleum Corp. The first indication of the Long Point salt dome was picked up by a German torsion balance crew working for the Gulf Production Co. but the seismograph more commonly gets the credit for the discovery of the dome, because it was used actually to establish the presence of the dome definitely. There had been no suspicion of the presence of a dome at that place, although it is on the edge of a large block of leases around some surficial sulfur water seeps of a type most commonly not considered as an indication of a salt dome. The Fannett salt dome was discovered by a Roxana Petroleum Corp. torsion balance crew and a German seismograph crew working for the Gulf Production Co., almost at the same time. Which had

actual priority in the discovery, the writer does not know, but as the Fannett salt dome happened to lie wholly within the block of Gulf Production Co. leases and wholly off the block of the Roxana Petroleum Corp'n. immediately to the south, the credit for the discovery of the dome usually is given to the former company and to the seismograph. The presence of either dome was not suspected until indication of the presence of a dome was given by the geophysical instruments. According to reports, the validity of which the writer does not know, indications of the presence of the Moss Bluff salt dome are now known to have been given by torsion balance surveys made by two different companies in areas closely adjacent to the dome, although they were not recognized in the interpretation of the surveys.

The presence of a deep salt dome has been indicated by the torsion balance near Dewalt, southwest of the Blue Ridge salt dome, in Fort Bend Co., Texas. The presence of the dome is reported to have been checked by the seismic method, but the prospect has not been drilled as yet.

A small oil field has been developed at the Nash salt dome and a sulfur deposit has been found that is slightly too deep to be mined profitably at present. The Long Point salt dome has given evidence of being a first class sulfur dome and a half interest in the sulfur rights has been sold by the Gulf Production Co. at a price (it is reported) that would pay several times over the cost of all the torsion balance work that has been done in the Gulf Coast. Two oil wells have been completed at the Allen dome and a small amount of oil has been found on the Fannett dome.

Some eight or ten torsion balance prospects in the Gulf Coast have been drilled and condemned. In no one of the cases known to the writer did the torsion balance indicate the presence of a dome with a probability of better than twenty chances out of one hundred. The Rycade Oil Corp'n. drilled three such prospects on the basis of torsion balance work done under the direction of the writer. At each of the prospects, there was some indication that in itself almost warranted drilling—three shallow water wells with authentic shows of oil at one prospect, a sulfur-water well similar to those at Nash at another, and a water well with a water of abnormal composition for surficial waters at the other—and at each prospect the torsion balance mapped a gravity high that was fainter than and not just like the gravity highs associated with the known salt domes. It was hoped that the gravity high mapped was the obscure trace of a deeply buried dome, but each of the prospects was recognized as being on the border line of favorability and unfavorability. A very distinct gravity minimum east of Welsh, La., mapped both by the Roxana Petroleum Corp'n. and by the Rycade Oil Corp'n. was considered by both companies as a prospect on the border line

between favorability and unfavorability. It was drilled without finding any indication of anything abnormal, but the test may not have been well located. The out and out failures of the torsion balance in the Gulf Coast area have been due, in the first place, to inexperience in interpretation; in the second place, to over-optimism in the use of indistinct indications; and in the third place, to the necessity of testing prospects of a lower order of favorability, in the absence of prospects of a high order of favorability.

Some fairly exact quantitative and semi-quantitative determinations have been made of the position and steepness of the flanks of the salt core and the cap and of the thickness and relative distribution of the cap. On account of its superior speed and its equal certainty of detection of a salt dome, above 2500 ft. and its superior probability of detecting deeper salt domes, and its ability to work marsh, swamp, and shallow lake areas with considerable speed, the seismic method is the best for use in reconnaissance for new salt domes; but after a relatively new salt dome has been discovered, the torsion balance has been giving superior results in detailing the dome. From surveys very carefully made with the purpose in view, it is possible to contour the top, flanks, and thickness of the cap rock of some domes with a high degree of accuracy and of other domes with a fair degree of accuracy.

Delimiting a Salt Dome from Torsion Balance Results

The predictions and the verification of such a quantitative survey of the Hoskins Mound salt dome are illustrated by Fig. 10. The upper two panels of the figure give the gradient profile and the gradient arrows of one of the several diametral sections that were run across the dome. The calculations were tied to the depth of the top of the cap and of the top of the salt in well A and to the top of the cap in several wells in the left half of the dome off the line of the section. The lower panel gives a vertical cross-section of the dome according to the calculations from the torsion balance observations. The wells B, C, D, and E were drilled after the calculations were made, and are the only wells in the right third of the dome; the point *TC* marks the top of the cap as it was actually found in each well. The actual against the predicted depths of the top of the cap for those wells are: Well B, actual 840 ft. (256.2 m.), predicted 900 (274.5 m.); Well C, actual 947 ft. (288.8 m.), predicted 1000 (304.8 m.); Well D, actual 1222 ft. (372.7 m.), predicted 1250 (381.3 m.); Well E, actual 1565 ft. (477.3 m.), predicted 1575 (480.4 m.).

The success and the degree of error in the delimitation of a dome entirely unknown except from the torsion balance survey have been illustrated by the discussions of the Nash dome and by Fig. 9. One difficulty about the exact checking of the torsion balance predictions is that

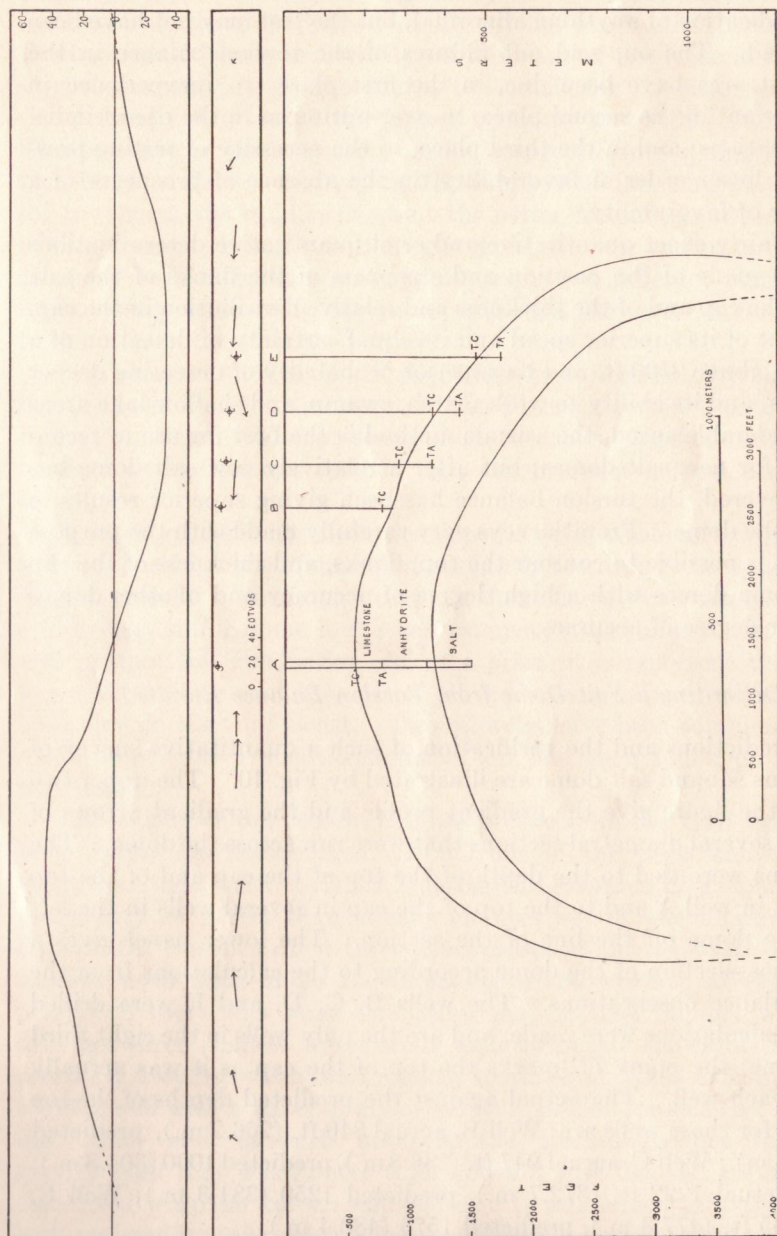


FIG. 10.—GRADIENT PROFILE, GRADIENT ARROWS AND CALCULATED STRUCTURAL SECTION FOR A DIAMETRAL LINE ACROSS THE HOSKINS MOUND SALT DOME, BRAZORIA COUNTY, TEXAS, SHOWING THE CHECK OF THE PREDICTIONS BY SUBSEQUENT DRILLING.

(Survey by the Geophysical Research Corp., Andrew Gilmour, observer and calculator under the direction of the writer. Published by the permission of P. George Maerky, Freeport Sulphur Co.)

they do not show minor irregularities and that actually the edge of the salt and cap and the top of the cap are very considerably irregular. Isolated wells, therefore, may not give a true picture of the conformation of the dome.

Mapping Faults with the Torsion Balance

The faults of the Somerset-Luling-Mexia-Powell-Sulphur River zone of faulting can be mapped with the torsion balance. Torsion balance profiles run under the writer's direction across the Sulphur River-Campbell fault, the Quinlan fault, the Powell, Richland, Currie, Wortham, Mexia, Luling fault-line oil fields, and faults southwest of Seguin in Guadalupe County and in southern Bexar County, indicated the presence and approximate position of the fault.

The indication by the torsion balance of the Luling fault is shown by Fig. 11. A single line of stations was run across the main part of the Caldwell County half of the field; Fig. 11a shows a map of that part of the field with the gradient arrows and the R lines for those stations; the upper panel of Fig. 11b gives the gradient profile for those stations and the lower panel gives the structure section along the same line (generalized after Brucks). This section shows somewhat close similarity to that of the ideal fault of Fig. 11b. Below a depth of about 2400 ft., limestone and schists are faulted against limestone and schist, except for the rather narrow band of the Trinity sand, and therefore present only a very slight density difference on the two sides of the fault. Between depths of 1600 and 2400 ft., the Edwards limestone is faulted up against the apparently lighter Eagleford, Del Rio and Austin and against the thin Buda of about the same density as itself; the Eagleford, Buda, Del Rio, and Austin are faulted up against the considerably lighter Taylor Navarro shales; and between 400 and 800 ft., the Navarro shales are faulted up against the slightly lighter Midway shales. The zone of the maximum gradient, therefore, lies over the zone of the trace of the fault in the Austin chalk, Del Rio shale, Buda limestone, Eagleford, Georgetown-Edwards limestone of the upthrow side rather than over the trace of the fault in the basement of the upthrow side, and on account of the dip of the fault to the left and the opposition of the heavier Navarro against the lighter Midway, the gradient profile is distinctly asymmetric and is fairly similar to the ideal profile of Fig. 8b. As the maximum near the right edge of the profile is based on the record of a single station, great dependence can not be placed upon it; it may signify a small fault or a concretion near the instrument, or some other surficial irregularity.

The faults of those zones have been mapped with the torsion balance by several other companies with varying degrees of success; whether the two or three failures have been failures of the method or of inexperienced interpreters, the writer is not sure. The faults in some places do not show

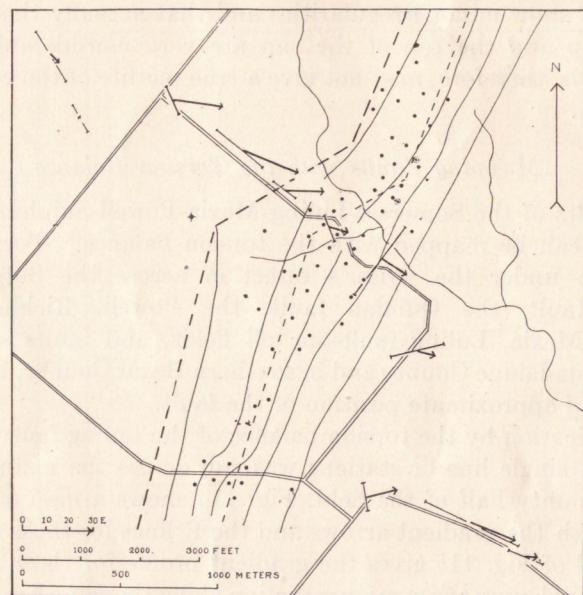


FIG. 11a.

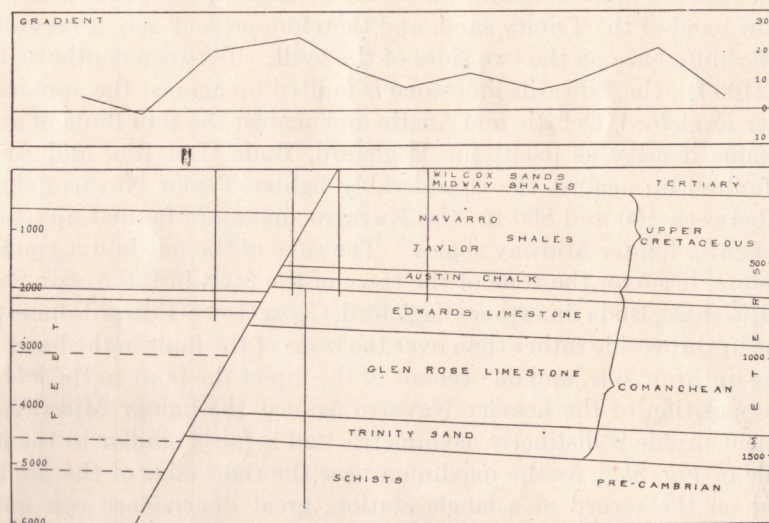


FIG. 11b.

FIG. 11.—EFFECT OF THE LULING FAULT ON THE GRAVITY GRADIENT.

a. Gradient arrow, "R" line, and structure map of a portion of the Luling (fault line) oil field.

b. Structure section and gradient profile.

(Survey by the Rycade Oil Corp'n. under the direction of the writer. Published by the permission of E. L. DeGolyer, President, Rycade Oil Corp'n.)

up in the torsion balance results as clearly as in others, and many situations would be rather difficult for an inexperienced and poorly trained man to interpret correctly.

In the Panuco district of Mexico, the torsion balance is reported to have done some rather brilliant work in making well locations along faults.

The torsion balance does much more brilliant work in detailing a fault than on reconnaissance for faults. As a single fault represents in most places a rather simple mathematical situation, rather brilliant results may be obtained in quantitative calculations of the fault, but as the gravitational effects of faults commonly are limited to a very narrow zone, stations must be placed very close together to map the fault. In reconnaissance, it is very easy to jump a fault, if any attempt is made to make speed in covering ground; and yet, taking stations every 500 ft. along a traverse across the supposed fault zone or into unknown territory is a tedious, time-consuming, and expensive reconnaissance. Furthermore, often faults are compound rather than clean-cut single breaks, and may bend or be offset or cut by cross-faults. All such irregularities can be worked and determined by taking a sufficient number of stations, but they greatly hinder an attempt to get a hasty idea of the situation by reconnaissance.

Work of Torsion Balance in Mid-continent Field

The work of the torsion balance in the Mid-continent district proper has been mostly reconnaissance to investigate the possibilities of the instrument and as yet very little drilling has been done on torsion balance structures. Although the torsion balance as yet has no Mid-continent oil field to its credit as an outright discovery, the work with the torsion balance in that area shows that it has very great possibilities. Buried granite, gneiss, and Cambro-Ordovician ridges such as those in the Panhandle, as the Hambro-Nocona-Bulcher-Muenster ridge, Healdton, the Criner Hills, and some of the Kansas granite ridges, show up brilliantly in torsion balance surveys. In southern Oklahoma, a study of three long reconnaissance profiles showed: (1) that the torsion balance results indicate seven first-class prospects and eleven second-class prospects; (2) that if two wildcats were drilled on each structure on locations made wholly from interpretation of the torsion balance results, two first-class oil fields, one second-class oil field, and one or two fourth-class oil fields would be discovered, and the results on two of the prospects would not be known; the efficiency per well for the discovery of oil fields would be 30 per cent., with 30 per cent. of the wells not heard from; (3) that if two wells were drilled similarly on each of the second-class prospects, the results would be the discovery of three third-class oil fields, one small gas field, and one barren structure; one structure would prove to be

false, and ten wells would be in areas where there has been no drilling; the efficiency per well for the discovery of oil fields would be 13 per cent. with the result of 45 per cent. of the wells not known.

Although in the Seminole district, the Seminole City is not very clearly indicated in a reconnaissance survey of the area, the Bowlegs field and the eastward-northward extension of the Earlsboro Wilcox pool were predicted from the torsion balance results, but in Sec. 2 T 7 S R 5 E, an A. P. C. well that is located on one of the better looking gravity highs of the area is apparently distinctly low. However, in the Seminole district, low wells are found rather anomalously in the middle of the structural highs and a single low well can not definitely condemn a surface or a gravity high. The Garber structure is faintly indicated by the torsion balance but on reconnaissance such a structure as Garber probably would not be detected unless the distance between the stations were 1500 ft. or less.

Survey of Buried Granite Ridge in Texas

The results of a reconnaissance torsion balance survey of the buried "granite" ridge of northwestern Cooke County, Texas, are shown in Fig. 12. The survey was of a sketchy reconnaissance type, comparable by analogy to a hasty Brunton-speedometer geologic reconnaissance for structure. The accuracy expected of it should be no greater than would be expected of such a hasty Brunton-speedometer reconnaissance; it should depict the high lights and major features of the structure but should not be expected to indicate accurately the details of the structure. The isogams shown in Fig. 12 are based on an approximately simultaneous least-square adjustment of the gravity differences between all the stations. A primary net of lines was laid out somewhat as in the primary net-method of Eötvös, but about one-third of the stations were used as key stations instead of merely six or seven as in the Eötvös method. This survey was tied into the Fox survey by three lines of traverse and it was possible, therefore, to give an absolute value to the isogams, which in this survey as in the Fox survey are referred to the level surface through the Busby station (U. S. C. and G. S. No. 305).

The indication of the structure is as accurate as could be expected from so sketchy a reconnaissance survey. Two wells have gone into schists within the area of the map, one a mile north of Muenster at a depth of 1570 ft. below sea level, and the other about six miles west of Bulcher, at a depth of 2013 ft. below sea level. Only two other wells within the area have gone to comparable depths: one was about five miles west of the second schist well and went to a depth of 2204 ft. below sea level without encountering pre-Pennsylvanian rocks, the other was about seven miles northeast of Muenster and encountered the Ellenburger limestone

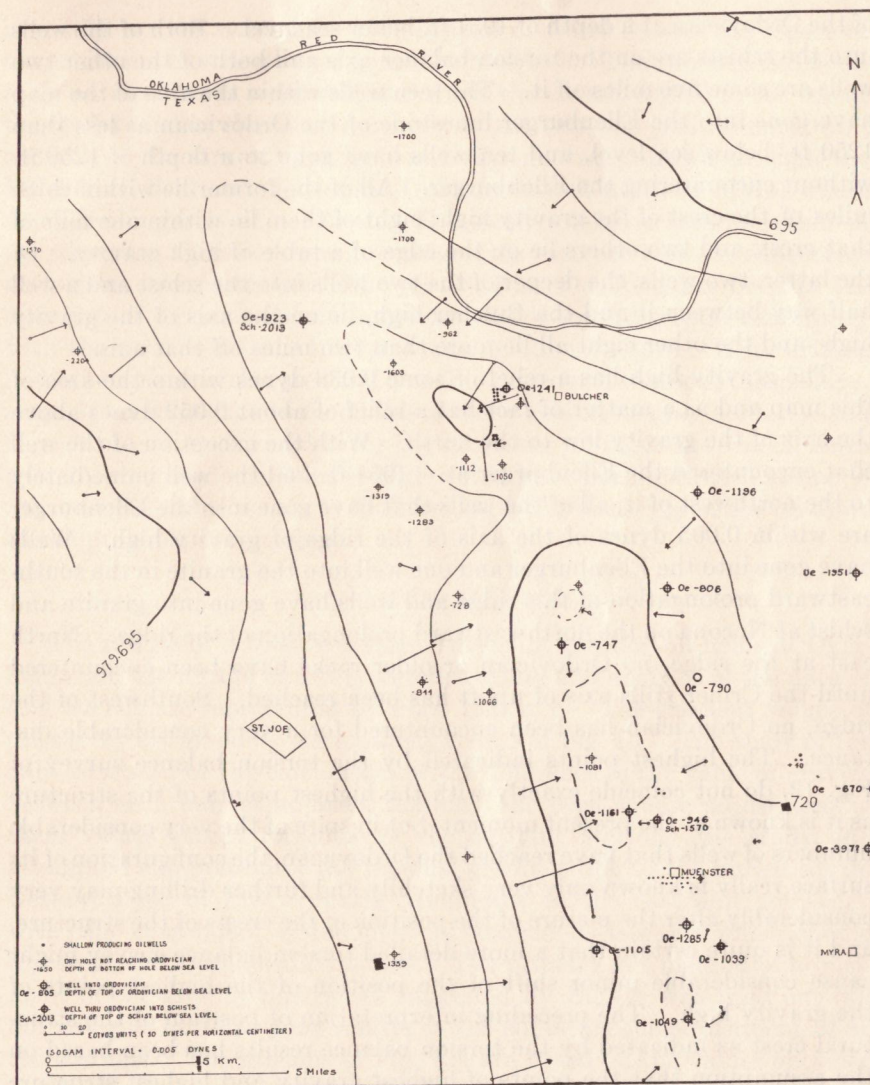


FIG. 12.—GRADIENT ARROW AND ISOGAM MAP OF THE MUENSTER-BULCHER "BURIED" ORDOVICIAN-METAMORPHIC RIDGE IN NORTHWESTERN COOKE COUNTY, TEXAS.

Isogams by the writer aided by I. R. and E. B. in a semi-simultaneous adjustment of the gravity differences between all stations. The isogams are tied into the U. S. Coast and Geodetic pendulum station No. 305, Busby, just south of the Fox oil field, Oklahoma, and are valid for the level surface through that station.

(Survey by the Geophysical Research Corp. under the direction of the writer. Published by permission of E. L. DeGolyer, President, Amerada Petroleum Corp.)

of the Ordovician at a depth of 1951 ft. below sea level. Both of the wells into the schists are on the torsion balance axis and both of the other two wells are some five miles off it. Thirteen wells within the area of the map have gone into the Ellenburger limestone of the Ordovician at less than 1250 ft. below sea level, and ten wells have gone to a depth of 1250 ft. without encountering the Ellenburger. All of the former lie within three miles of the crest of the gravity high; eight of them lie within one mile of that crest, and two others lie on the edge of a table of high gravity. Of the latter, two wells, the deeper of the two wells into the schist and a well half way between it and the Bulcher high, lie on the axis of the gravity high; and the other eight all lie more than two miles off that axis.

The gravity high has a relief of some 0.038 dynes within the area of this map and as a matter of fact has a relief of about 0.052 dynes above the axis of the gravity low to the north. With the exception of the well that encountered the Ellenburger at -1951 ft. and the well immediately to the northwest of it, all of the wells that have gone into the Ellenburger are within 0.005 dynes of the axis of the ridge of gravity high. Wells have gone into the Ellenburger and one well into the granite in the south-eastward prolongation of this ridge and wells have gone into granite and schist at Nocona on the northwestward prolongation of the ridge. North east at the ridge, no Ordovician or older rocks have been encountered until the Criner Hills axis of uplift has been reached. Southwest of the ridge, no Ordovician has been encountered for a very considerable distance. The highest points indicated by the torsion balance survey of Fig. 12, do not coincide exactly with the highest points of the structure as it is known at the present moment, but in spite of the very considerable numbers of wells that have reached the Ordovician, the configuration of its surface really is known only very sketchily and further drilling may very considerably alter the picture of the position of the crests of the structure, and it is quite certain that a more detailed torsion balance survey might cause considerable minor shift of the position of the highest points of the gravity high. The preceding interpretation of position of the structural crest as indicated by the torsion balance results has been based on the assumption that the points of highest gravity and highest structure coincide. That is not necessarily true but in the area under discussion is approximately true. A more detailed analysis of the torsion balance results than the writer has had time to make might show that the structural crest as indicated by the gravity relations should be shifted a mile more or less to the northeastward of the crest of the gravity high.

The survey of this area was made at a time when little was known about the Ordovician of the area. The well into the Ordovician just west of Bulcher and the two wells into the Ordovician at -1196 and -1951 ft. due southeast of Bulcher had just been drilled; the depth of the top of the Ordovician in the well immediately north of Muenster was rather

generally carried as about -1500 instead of -946 ft. and the report of the schist in the well was not generally believed. The two wells immediately north of Myra were well known but the data in regard to the top of the Ordovician in the more southerly of the two was (and is) uncertain; the axis of the structural ridge was rather generally drawn through the Bulcher oil field and the second Ordovician well north of Myra, where the Ellenburger is at -670 ft. The torsion balance interpretation of the structure, therefore, was at considerable variance with the subsurface geologists' interpretation. The result of later drilling has been in the direction of a revision of the subsurface maps toward greater conformity with the torsion balance interpretation of the structure. If no drilling had taken place in this area and if nothing were known about the geology, acreage taken on the basis of this sketchy reconnaissance survey would be well placed, and would cover the Muenster oil fields and the oil fields near Bulcher; and the Bulcher high would have been indicated as one of the most favorable points to drill first, with the discovery of the oil there as the probable consequence.

Survey of Fox Oil Fields

The results of a torsion balance survey of the Fox oil fields of Oklahoma and the results of a very detailed study and analysis of the data of that survey are represented by Figs. 13a, b, c. The gradient values of the survey are given by the gradient arrows of Fig. 13. The study of the data consisted of the calculation of the difference in the value of gravity between each pair of adjacent stations, the least-square adjustment of these values except for some of the outlying stations, the study and attempted elimination of the regional gradient due to the buried southwestern slope of the Arbuckle Mountains to the east and northeast, with the consequent obtainment of anomalous isogams which should represent the more local structure. The primary net method of adjustment of the gravity differences suggested by Eötvös was tried out but did not give quite as good results as the simultaneous adjustment of all the stations. The isogams of Fig. 13a are based on the values obtained by the simultaneous least-square adjustment of the 68 stations lying south and west of Fox, and by the linear adjustment of closed traverses to the north, east and southeast of Fox. The survey was tied into the Busby pendulum station of the U. S. Coast and Geodetic Survey, which made it possible to give absolute values to the isogams; in this case their value represents the value of gravity in the level surface through the Busby station. By using a correction based on the difference in elevation between any point within the area of the survey and the Busby station, the approximate actual value of the gravity at that point can be determined from the isogams.

Indication of the Fox oil-field structures is only indifferently visible in the gradient arrows and isogams of Fig. 13a. The general north-northeasterly trend to the gradient and the general increase north-northeastward of the value of gravity are the effects of the Arbuckle Mountains lying not far to the east and northeast and of the buried southwestward slope of the Arbuckle mass. A slight indication of underlying of structure of the Fox oil field can be seen in the small size of the gradient arrows

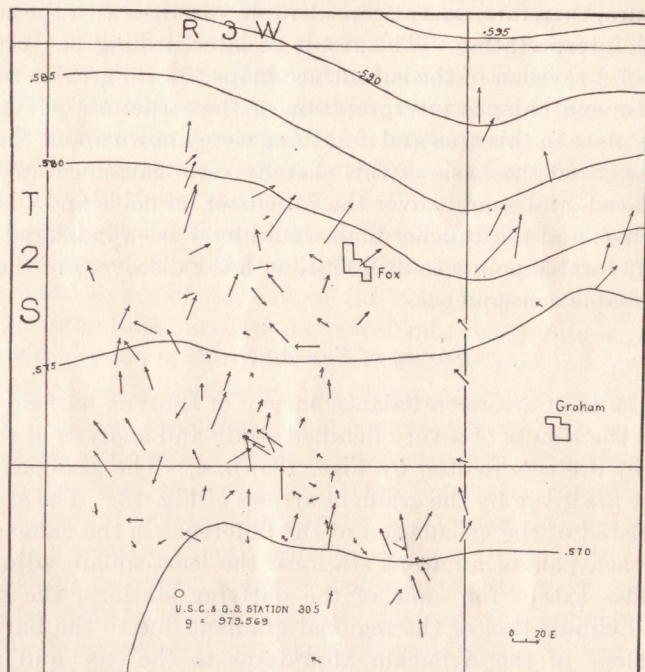


FIG. 13.—ANALYSIS OF A TORSION BALANCE SURVEY OF THE FOX OIL FIELDS, OKLAHOMA.

(Survey by the Amerada Petroleum Corp., simultaneous least-square adjustment of the gravity differences between all stations by I. Roman, analysis by the writer. Published by the permission of E. L. DeGolyer, President, Amerada Petroleum Corp.)

a. Gradient arrow and isogam map. The isogams are tied to the Busby Station, U. S. Coast and Geodetic Survey pendulum station No. 305, and are referred to the level surface through that station.

and a tendency to reversal or rotation of their orientation about 1.5 mile southwest of Fox. As the effect of the Fox structure seemed to be so obscured by the effects of the Arbuckle mass, a further study and analysis of the situation was made with the purpose of eliminating the effect of the Arbuckle mass. The anomalous isogams of Figs. 13b and 13c represent the variation of gravity, which apparently is due to more local structural anomalies of density than the Arbuckle mass. The isogams of the two figures are the same but in Fig. 13b they are superimposed on the Bureau of Mines structure contour map of the Fox oil fields and in

Fig. 13c they are superimposed on the structure contours of a generalized A. P. C. structure contour map of the area surrounding the oil fields. The anomalous isogams give a rough picture of the structure; the agreement of the anomalous isogam map to each of the two structure maps is about of the same order as the agreement between the two structure contour maps.

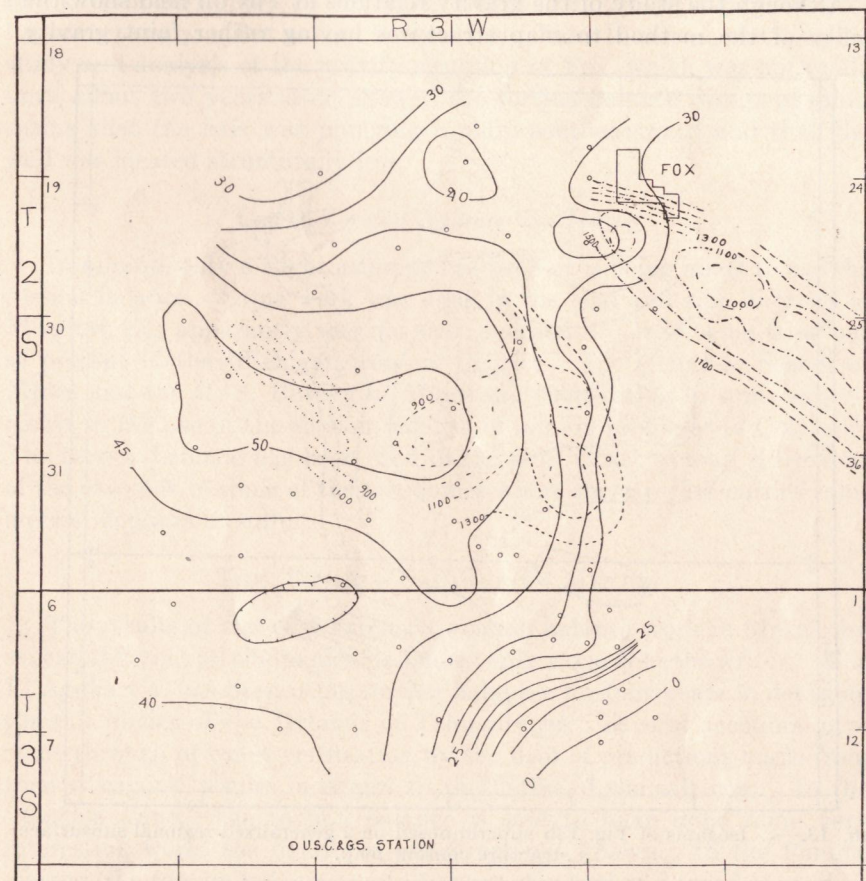


FIG. 13.—b. Map of the local anomalous isogams superimposed on a U. S. Bureau of Mines structure contour map of the Fox and Graham oil fields. The datum horizon is different in each of the three oil fields.

If this had been a wildcat prospect and acreage had been taken on the basis of the torsion balance results, a considerable part of the acreage would have been within the area that has proved to be productive. The slight shift of the anomalous isogams southwest from the apparently corresponding structure is probably due to a slight error in the estimation of the magnitude of the regional gradient due to the Arbuckle mass.

At the time that the study was made, the Bureau of Mines map was the only one available to the writer and the indication by the torsion balance of a ridge running off to the northwest was thought to be a "bust," it was not until considerably later that the A. P. C. map was received, showing that there was geological indication of a structural ridge extending off in that direction.

Although the study of the gravity relations at Fox oil field show the ability of the method to map structures having rather faint gravity

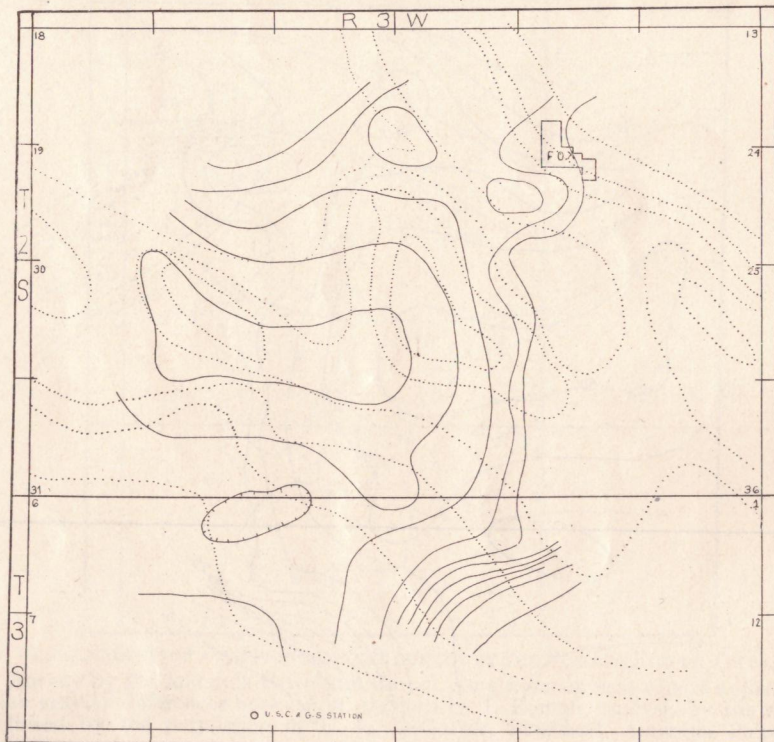


FIG. 13.—c. Isogams of Fig. 13b superimposed on a generalized regional subsurface structure contour map.

effects, great dependence on such faint indications of structure should not be made in reconnaissance work; the type of analysis that was made at Fox was warranted only by the very complete net of stations which cover the area with a much greater thoroughness than would be warranted in mapping most wildcat prospects. In the absence of geological indications of the structure, such faint indications will be better than nothing, but in a great many cases, the indications will prove false or the position of the structure will be faultily located.

A mistake that will have to be debited against the torsion balance, the writer, and insufficient experience, was made at Fox. An axis with

large gradient arrows on the southwest and small and abnormally oriented gradients on the northeast can be seen running northwest-southeast through the center of the south side of T 2 S 3 W in Fig. 13a. The axis was interpreted as indicating the southeastward extension of the sharp structure of northwest Fox oil field and as a result of our interpretation the Amerada Petroleum Corp. drilled a well in section 2 T 3 S R 3 W not far from the upper "25" of Fig. C, but the well proved to be extremely low structurally. But as can be seen from Figs. 13 b or c, the detailed study and analysis of the gravity situation at Fox, which was not made until about two years later, showed the torsion balance results as indicating that the axis was plunging rapidly southeastward and that the well was located structurally low.

Use of Torsion Balance in Mining

In mining, only a slight attempt has been or is being made to use the torsion balance. Some work was done in the lead and zinc districts of Missouri, but apparently was not very successful. It is being tried out at present in the Michigan iron-ore district. The Colorado School of Mines and the U. S. Bureau of Mines are coöperating in some experiments in the use of the torsion balance in mining problems in Colorado. The torsion balance has been used in the Gulf Coast to map the extent of the cap rock in some of the salt domes where the cap rock carries commercial deposits of sulfur.

THE TORSION BALANCE IN MEXICO

The results of the very extensive torsion balance work in Mexico by several different oil companies are known only vaguely to the writer. The El Aguila Co. has been using torsion balances for four years in detailing the salt domes of the Isthmus of Tehuantepec. Several accounts have come through of exact verification by the drill of predictions made from torsion balance results in regard to the flanks of the salt core. In the "Golden Lane," the torsion balance is said to have done some very interesting work, the details of which are not known. In the Panuco district, the torsion balance is said to have shown that a trend of production which unexplainably lay at an angle to the strike of the surface structure coincided with the crest of a buried ridge in the basement complex. A brilliant piece of work is reported in the location of faults in the Panuco district.

THE FUTURE OF THE TORSION BALANCE

A distinct parallelism holds between the relation of the gravimetric surveys to geologic structure and the relation of structural geologic surveys to the occurrence of oil. As there is a tendency to a parallelism

between the occurrence of oil and of certain types of structural situations, geologic structure may be used as an indirect method of finding oil. As there is similarly a tendency to a parallelism between geologic structure and the anomalous distribution of density, gravimetric surveys with the torsion balance may be used as an indirect method of mapping geologic structure. As with oil geology, the torsion balance can handle certain types of situations brilliantly, does fairly good work in many others, and can do nothing at all in some cases. The torsion balance, like all the other geophysical instruments, is not a panacea for hunting geologic structure and can not replace geology, where good geologic work is possible; it should be used to supplement geology, in areas where only a small amount of structural work can be done, or to fill in the structure in areas where no structural work is possible. The success of the torsion balance in locating and defining oil structures will be just about the same as the success of geology in finding oil.

Use of the Balance in Oil Work

In the use of the torsion balance in oil work, four facts should be remembered: (1) that what the torsion balance maps is differences in density; (2) that the number of stations necessary to map a structure with a certain degree of accuracy is independent of the size of the structure; (3) that surficial and topographic irregularities of which the effects can not be calculated limit torsion balance surveys; and (4) that the interpretation of geologic structure from gravimetric surveys is just as complicated a subject as the interpretation of the probable occurrence of oil from a structural geologic survey.

The torsion balance may be used either for reconnaissance or detail mapping but in planning a program of work the possibilities and limitations of the method, as well as the importance of working some particular area, must be kept in mind. The method will give the most brilliant results where it is used in mapping structures that have produced considerable and sharp contrasts in density—for example, salt domes, granite ridges, faults in which massive limestone or anhydrite are faulted against sands and clays—and it will produce those results with a wider spacing of stations on large structures than on small. Where prolific production is obtained from structures that do not produce much of a density contrast and that are not reflected in the surface geology, it may be well worth while to attempt to get some clue to the structure through torsion balance surveys, but the results will not be so brilliant or so reliable as the preceding cases; and it will require an extensive survey with a closely spaced set of stations and an analysis of results of a much higher grade of technique and experience in interpretation. The practicability of torsion balance work is limited by surficial irregularities of

mass in the upper subsoil and by rugged topographic relief, but if the necessities warrant the expense, an area of very considerable surficial irregularities of mass can be worked by occupying an enormously increased number of stations very closely spaced and by using a least-square adjustment to smooth out the irregularities. If the necessities warrant, an area of fairly rugged relief often could be worked by making an extensive survey of the topography and of the densities of the formations and then calculating the effects of the topography on the gradient and curvature. Except in mountainous or semi-mountainous regions, the effects of topographic irregularity are not so serious as is somewhat commonly believed, if the gravity anomalies due to the structure to be mapped are at least moderately large in size.

The present status of the torsion balance method for most oil companies is about the same as if the companies had to depend on young civil engineers for their geology. In a region of moderate relief where an easily identifiable limestone keybed can be walked out around the hill-sides, a young civil engineer would be able to make an accurate plane-table map of the surface structure. If he had a hazy idea of the anticlinal theory of the occurrence of oil, he would be able to make a fairly good guess where to drill on well defined anticlines and domes, but his naive interpretation of more complicated structural situations would be badly in error. Torsion balance observations are rather simple affairs. The visual types of instruments are simpler to operate than an explorer's alidade and, with the simple tabular computation forms, the calculations are reduced to a routine of elementary addition, subtraction, and multiplication. A bright young man without technical education can be trained to be an efficient torsion balance operator (for the Süss type of balance) more easily than he can be trained to be an efficient plane-table instrumentman. Such an observer normally will be able to bring an accurate map of the gradient and curvature of an area, but neither he nor a geologist who has not studied the interpretation of torsion balance results will be able to make an accurate interpretation of his results. They would be able to make a fairly good, approximate qualitative interpretation of the survey of some salt domes and some buried granite ridges. In a survey of the Healdton field, they would recognize the presence of some large, sharp structure but would interpret its form and position incorrectly. In surveys of Fox-Graham or of Garber, they would not recognize the presence of the structures.

Qualifications of an Interpreter of Torsion Balance Results

The difficulty is that the indirect relation between geologic structure and the distribution of density is more intricate and complex than the indirect relation between geologic structure and the accumulation of oil.

A torsion balance survey for practical purposes is an indirect method of mapping the distribution of density in the subsurface, and mapping the distribution of density in the subsurface is only an indirect method of mapping structure. Therefore the interpreter of the torsion balance surveys must know: (1) the mathematical technique of interpreting bodies from the distributions they produce of gradient and curvature values; (2) the relationship between geologic structure and the distribution of density; and (3) the probabilities, possibilities, and impossibilities in the way of occurrence of geological conditions—and, not least, he should have a level-headed sense of the limitations and imperfections of the method.

To get the most out of torsion balance surveys, the interpreter should be a good mathematician and geologist, should have had a chance to study the mathematics of interpretation and should have had a broad practical experience with the method. There are very few men who have had sufficient opportunity to study the mathematics of the interpretation and who have had sufficiently broad experience to qualify as competent interpreters. It is, and will be, easy to get men who are able and competent observers and who possess a fair but superficial knowledge of interpretation. A great danger to the full realization of the possibilities of the method is that the company employing such a man as an interpreter may fail to recognize his limitations. Another danger is that a company that realizes the necessity of training itself and its interpreter by theoretical study and practical experience will become overimpatient and over-enthusiastic and may force him to practical utilization of the method before he is competent to use it. A company that has had much experience with geology has become educated to the fact that geology is not a "sure-shot" panacea for finding oil, and is satisfied to allow geology a considerable number of failures if it turns up a fair number of successes; but there is a good possibility that the same company will condemn the torsion balance after a very few failures, without regard to whether the failures were due to the method or to the incompetence or inexperience of the interpreter.

Conflict between the gravimetric and the geological interpretations of the same structural situation will inevitably arise in the course of the torsion balance surveys. In some cases, the conflict will be due to the incomplete concordance of structure and the distribution of density, but in many cases the geologic interpretation may not be completely accurate. In the absence of better data over wide areas, the geologist has to depend on those from widely scattered wells and often on the lithologic log to log correlation of drillers' logs not checked by geologists or paleontologists. Although such correlation may be necessary and useful, it holds grave possibilities of error, and although generalization from the scattered wells also may be necessary and useful, it should be recognized and remembered

as a first and very rough approximation to the truth. Because he is forced continually to use more or less inaccurate and inadequate data to make conclusions that must be acted on as if accurate, the geologist tends to become callous, and to forget the probably considerable degree of inaccuracy of his conclusions, and, in judging the merits of a conflict between the geological and the torsion balance interpretations of a structural situation, he will tend to overvalue the validity of the geologic interpretation and undervalue the validity of the torsion balance interpretation. For example, if the surface evidence of the Healdton (Okla.) structure were effectually concealed beneath a mantle of Trinity sand, and if the only wells drilled in the area were a deep test some two miles northeast of the northeast edge of the field and another deep test some four miles southwest of the southwest edge of the field, most geologists familiar with the area, but unfamiliar with the torsion balance, would condemn the torsion balance indication of a big structure out in the middle of a very deep Red Bed syncline as an impossibility and a bad "bust" on the part of the torsion balance method or the geophysicist. If therefore the torsion balance gives a definite indication of some type of structure widely divergent from that indicated by geology, the probable accuracy of the geologic interpretation must be reevaluated. The conflict between the geologic and torsion balance interpretation of some structural situation in many cases will be analogous to the proverbial conflict in the description of an elephant by three blind men; one of whom felt the trunk, the second the side of the elephant, and the third a tusk.

Combination of Geophysical Instruments Desirable

Although the torsion balance is superior to the other types of geophysical instruments in doing certain types of work in certain types of geologic situations, it is inferior to some of the geophysical instruments for other purposes or in other types of situations, and in many cases it is preferable to combine several geophysical methods. In the salt dome area of the Texas-Louisiana Gulf Coast, the mirage elastic earth-wave method is superior to the torsion balance method for reconnaissance for salt domes, on account of greater speed and ability to cover difficultly leasable or passable acreage, lower cost and equal accuracy in detecting the shallower domes and greater accuracy in detecting the deeper domes; but the torsion balance is superior for detailing the shallower domes and possibly certain types of the deeper domes. If the structural deformation is slight or if it does not cause much deformation of the distribution of density, and if a moderately thick hard bed lies at moderate depth under a cover of much softer beds, the reflection elastic earth-wave method may be far superior in accuracy, speed, and cost. Although the results of magnetometer surveys can not be depended on to the same extent

as those of torsion balance surveys, the magnetometer does give indication of certain types of structure in a great many places, and as it is faster and less costly to run than the torsion balance, it can be used advantageously on reconnaissance to discover areas to test with the torsion balance.

Usefulness of Torsion Balance in Pure Science

In connection with the pure-science problems of geology, the future seems to offer possibilities of usefulness for the torsion balance. The reconnaissance survey already made across the buried "granite" ridges of the Mid-continent indicate that regional reconnaissance of such sedimentary basins will reveal the structure and the structural trend lines of the basement and will allow buried mountain systems to be traced out into and across such basins. The prolongation of the Appalachian mountain system after it plunges out of sight under the sedimentary cover of the Gulf Coastal Plain comes up frequently in the broader philosophic discussions of the orogenesis of the American continent and allied problems. Although the geologic map of Alabama does not seem to indicate that the mountains may swerve from a course that will carry them straight out under the Coastal Plain into the Gulf of Mexico, it is not uncommon to find the argument that they bend westward to connect with the late Paleozoic mountains of Arkansas and Oklahoma. The torsion balance would probably be able to give some very brilliant results in proving once and for all what becomes of the Appalachian Mountains under the Coastal Plain of Alabama, and whether they dip gently under the Coastal Plain sediments out into the Gulf of Mexico or are abruptly faulted off.

In the discussions of isostasy and the questions whether or not certain areas or structural units are isostatically compensated or supported by the rigidity of the crust, arguments are used that in part depend on gravity anomalies observed at rather scattered and in many cases isolated pendulum stations. From those stations it is not possible to determine the depth of the masses that are producing the anomalies, but it has been suspected that many of them are relatively shallow. Many gravity anomalies have been mapped in regional reconnaissance with the torsion balance that are relatively local in area, that are demonstrably due to relatively shallow anomalies of mass, and that are of the same magnitude as the anomalies observed at the pendulum stations. Surveys made by the torsion balance in conjunction with the pendulum observations will allow a determination of the approximate order of depth or depth of the greater per cent. of the anomalies of relatively shallower mass and will make it possible to determine whether or not the anomaly observed at a pendulum station is due to a local cause or a regional one.

The batholith is one of the liveliest and most discussed problems of petrology, and the petrologists are keenly interested in extrapolating from the surface data down into the interior of the batholith to determine

its extent and composition. If a batholith is in a region that is not too complicated structurally or lithologically or where the surface relief is not too great—for example, Dartmoor, in England—there seems to be a reasonable chance that a careful torsion balance survey would reveal some very interesting data regarding the form, depth and composition of the batholith. The relative proportions of limestone and igneous rock in a coral island are reported to have been studied by a Japanese geologist by means of a torsion balance survey. The glacial geologist who is interested in mapping pre-glacial drainage systems should be able to get some brilliant results with the torsion balance, especially in areas where the bed rock is metamorphic, igneous, or limestone. For a wide range of problems in which the relations of rocks of abnormally high or low density are involved, the torsion balance is an instrument for looking down into the earth's crust, as it were, as it is possible now only in a scattered and very limited way by shafts, drill holes, and canyons.

REFERENCES

- The original and still the most fundamental references on the Eötvös torsion balance are:
- Roland v. Eötvös: Untersuchungen über Gravitation und Erdmagnetismus. *Ann. der Phys. und Chem.* [N. F.] (1896) 59.
- Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflächen mit Hilfe der Drehwage. Verh. d. XV, allgemeinen Konferenz d. internationalen Erdmessung in Budapest, 1906.
- Geodätische Arbeiten in Ungarn, besonders über Beobachtungen mit der Drehwage. Verh. d. XVI, allgemeinen Konferenz d. internationalen Erdmessung in London u. Cambridge, 1909.
- Die Niveauflächen und die Gradienten der Schwerkraft, am Balatonsee. Budapest, 1908.
- The more important papers in English are:
- Stephen Rybar: The Eötvös Torsion Balance and Its Application to the Finding of Mineral Deposits. *Econ. Geol.* (1923) 18, 639-662.
- H. Shaw and E. Lancaster-Jones: The Eötvös Torsion Balance. *Proc. Phys. Soc. of London* (1923) 35, 151-165.
- The Application of the Eötvös Torsion Balance to the Investigation of Local Gravitational Fields. *Ibid.* (1923) 36.
- The Eötvös Torsion Balance and Its Application to the Location of Minerals. *Mining Magazine* (London) (Jan.-Feb., 1925) 18-26, 86-93.
- W. Herbert Fordham: Oilfinding by Geophysical Methods. *Jnl. Inst. Pet. Tech.* (London) (October, 1925).
- Also reprinted and distributed by L. Oertling Ltd., London.
- Anonymous: The Eötvös Torsion Balance. A pamphlet to be used as a manual with the Oertling torsion balance. L. Oertling Ltd., London, 1925.
- H. Shaw and E. Lancaster-Jones: Locating Minerals and Petroleum. Being a Series of Articles on the Theory and Practical Use of the Eötvös Torsion Balance, reprinted from the *Mining Magazine* (London) (April to July, 1927). L. Oertling Ltd., London, 1927.